ELECTRICAL CIRCUITS

LECTURE NOTES

B.TECH
(I YEAR – II SEM)
(HM23)
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Department of Electrical and Electronics Engineering

ANNAMACHARYA INSTITUTE OF TECHNOLOGY & SCIENCES

(Autonomous Institution – UGC, Govt. of India)

I B.Tech EEE II Sem
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ELECTRICAL CIRCUIT ANALYSIS -I (EEE)

Course Objectives: To develop an understanding of the fundamental laws, elements of electrical   circuits and to apply circuit analysis to DC and AC circuits.

Course Outcomes:
1. CO1: Remembering the basic electrical elements and different fundamental laws.
2. CO2: Understand the network reduction techniques, transformations, concept of self-inductance and mutual inductance, phasor diagrams, resonance and network theorems.
3. CO3: Apply the concepts to obtain various mathematical and graphical representations.
4. CO4: Analyze nodal and mesh networks, series and parallel circuits, steady state response, different circuit topologies (with R, L and C components).
5. CO5: Evaluation of Network theorems, electrical, magnetic and single-phase circuits.

UNIT I INTRODUCTION TO ELECTRICAL CIRCUITS Basic Concepts of passive elements of R, L, C and their V-I relations, Sources (dependent and independent), Kirchoff's laws, Network reduction techniques (series, parallel, series - parallel, star-to-delta and delta-to-star transformation), source transformation technique, nodal analysis and mesh analysis to DC networks with dependent and independent voltage and current sources, super node and super mesh analysis.

UNIT II MAGNETIC CIRCUITS Basic definition of MMF, flux and reluctance, analogy between electrical and magnetic circuits, Faraday's laws of electromagnetic induction – concept of self and mutual inductance, Dot convention – coefficient of coupling and composite magnetic circuit, analysis of series and parallel magnetic circuits.

UNIT III SINGLE PHASE CIRCUITS Characteristics of periodic functions, Average value, R.M.S. value, form factor, representation of a sine function, concept of phasor, phasor diagrams, Concept of power factor, real power, reactive power, J Notation, Complex and Polar Forms of representation. Steady state analysis of R, L and C circuits to sinusoidal excitations-response of pure resistance, inductance, capacitance, series RL circuit, series RC circuit, series RLC circuit, parallel RL circuit, parallel RC circuit.

UNIT IV RESONANCE AND LOCUS DIAGRAMS Series Resonance: Characteristics of a series resonant circuit, Q-factor, selectivity and bandwidth, expression for half power frequencies; Parallel resonance: Q-factor, selectivity and bandwidth; Locus diagram: RL, RC, RLC with R, L and C variables.

UNIT V NETWORK THEOREMS (DC & AC EXCITATIONS) Superposition theorem, Thevenin's theorem, Norton's theorem, Maximum Power Transfer theorem, Reciprocity theorem, Millman's theorem and compensation theorem

Textbooks:
1. Engineering Circuits Analysis, Jack Kemmerly, William Hayt and Steven Durbin, TataMc Graw Hill Education, 2005, sixth edition.
2. Network Analysis, M. E. Van Valkenburg, Pearson Education, 2019, Revised ThirdEdition

Reference Books:
1. Fundamentals of Electrical Circuits, Charles K. Alexander and Mathew N.O. Sadiku,Mc Graw Hill Education (India), 2013, Fifth Edition
2. Electric Circuits (Schaum's outline Series), Mahmood Nahvi, Joseph Edminister, and K. Rao, Mc Graw Hill Education, 2017, Fifth Edition.
3. Electric Circuits, David A. Bell, Oxford University Press, 2009, Seventh Edition.
4. Introductory Circuit Analysis, Robert L Boylestad, Pearson Publications, 2023, Fourteenth Edition.
5. Circuit Theory: Analysis and Synthesis, A. Chakrabarti, Dhanpat Rai & Co., 2018,Seventh Revised Edition.

Web Resources: 1. https://onlinecourses.nptel.ac.in/noc23_ee81/preview 2. https://nptel.ac.in/courses/108104139 3. https://nptel.ac.in/courses/108106172 4. https://nptel.ac.in/courses/117106108

UNIT I INTRODUCTION TO ELECTRICAL CIRCUITS

INTRODUCTION:

An Electric circuit is an interconnection of various elements in which there is at least one closed path in which current can flow. An Electric circuit is used as a component for any engineering system.

The performance of any electrical device or machine is always studied by drawing its electrical equivalent circuit. By simulating an electric circuit, any type of system can be studied for e.g., mechanical, hydraulic thermal, nuclear, traffic flow, weather prediction etc.

All control systems are studied by representing them in the form of electric circuits. The analysis, ofany system can be learnt by mastering the techniques of circuit theory.

The analysis of any system can be learnt by mastering the techniques of circuit theory.

Elements of an Electric circuit:

An Electric circuit consists of following types of elements.

Active elements:

Active elements are the elements of a circuit which possess energy of their own and can impart it toother element of the circuit.

Active elements are of two types

        a) Voltage source   b) Current source

A Voltage source has a specified voltage across its terminals, independent of current flowing throughit.
A current source has a specified current through it independent of the voltage appearing across it.

Passive Elements:

The passive elements of an electric circuit do not possess energy of their own. They receive energy from the sources. The passive elements are the resistance, the inductance and the capacitance. When electrical energy is supplied to a circuit element, it will respond in one and more of the followingways.

If the energy is consumed, then the circuit element is a pure resistor.

If the energy is stored in a magnetic field, the element is a pure inductor.

And if the energy is stored in an electric field, the element is a pure capacitor.

Linear and Non-Linear Elements.

Linear elements show the linear characteristics of voltage & current. That is its voltage-current characteristics are at all-times a straight-line through the origin.

For example, the current passing through a resistor is proportional to the voltage applied through its and the relation is expressed as V or $V = IR$. A linear element or network is one which satisfies the principle of superposition, i.e., the principle of homogeneity and additivity.

Resistors, inductors and capacitors are the examples of the linear elements and their properties do not change with a change in the applied voltage and the circuit current.

Non linear element's V-I characteristics do not follow the linear pattern i.e. the current passing through it does not change linearly with the linear change in the voltage across it. Examples are the semiconductor devices such as diode, transistor.

Bilateral and Unilateral Elements:

An element is said to be bilateral, when the same relation exists between voltage and current for the current flowing in both directions.

Ex: Voltage source, Current source, resistance, inductance & capacitance.

The circuits containing them are called bilateral circuits.

An element is said to be unilateral, when the same relation does not exist between voltage and current when current flowing in both directions. The circuits containing them are called unilateral circuits.
Ex: Vacuum diodes, Silicon Diodes, Selenium Rectifiers etc.

Lumped and Distributed Elements

Lumped elements are those elements which are very small in size & in which simultaneous actions takes place. Typical lumped elements are capacitors, resistors, inductors.

Distributed elements are those which are not electrically separable for analytical purposes.

For example a transmission line has distributed parameters along its length and may extend for hundreds of miles.

The circuits containing them are called unilateral circuits.

Types of Sources:

Independent & Dependent sources:

If the voltage of the voltage source is completely independent source of current and the current of the current source is completely independent of the voltage, then the sources are called as independent sources.

The special kind of sources in which the source voltage or current depends on some other quantity in the circuit which may be either a voltage or a current anywhere in the circuit are called Dependent sources or Controlled sources.

There are four possible dependent sources:

        a. Voltage dependent Voltage source
        b. Current dependent Current source
        c. Voltage dependent Current source
   d. Current dependent Current source
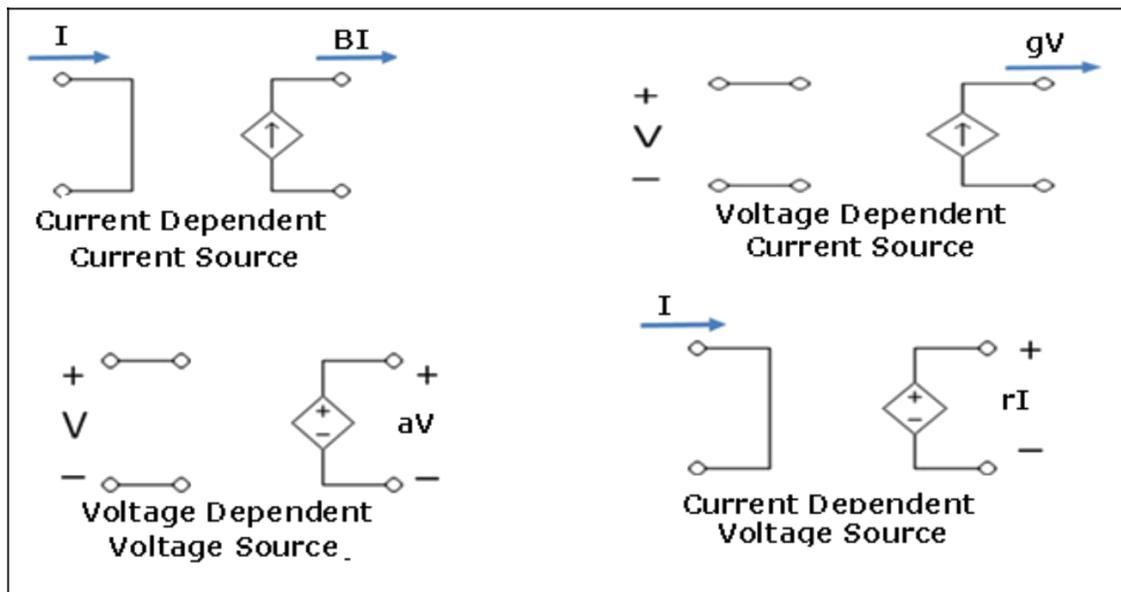
The constants of proportionalities are written as B, g, a, r in which B & a has no units, r has units of ohm & g units of mhos.

Independent sources actually exist as physical entities such as battery, a dc generator & an alternator. But dependent sources are used to represent electrical properties of electronic devices such as OPAMPS & Transistors.

Ideal & Practical sources:

1. An ideal voltage source is one which delivers energy to the load at a constant terminal voltage, irrespective of the current drawn by the load.

2. An ideal current source is one, which delivers energy with a constant current to the load, irrespective of the terminal voltage across the load.

3. A Practical voltage source always possesses a very small value of internal resistance r. The internal resistance of a voltage source is always connected in series with it & for a current source; it is always connected in parallel with it. As the value of the internal resistance of a practical voltage source is very small, its terminal voltage is assumed to be almost constant within a certain limit of current flowing through the load.

4. A practical current source is also assumed to deliver a constant current, irrespective of the terminal voltage across the load connected to it.
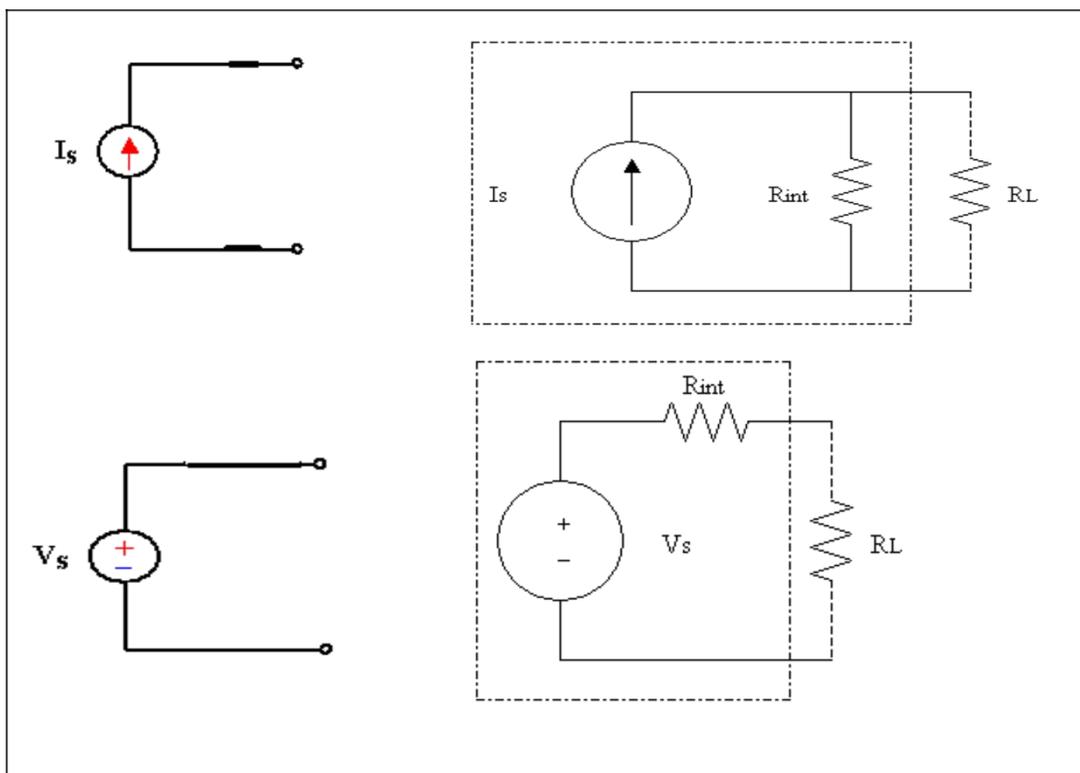
**Ideal voltage source connected in series:**
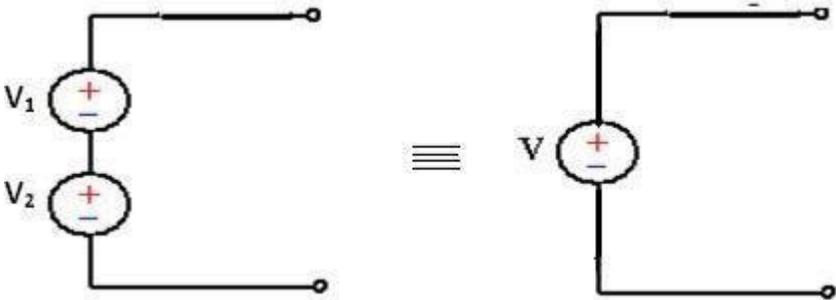
The equivalent single ideal voltage some is given by   V= V1 + V2

Any number of ideal voltage sources connected in series can be represented by a single ideal voltage some taking in to account the polarities connected together in to consideration.

Practical voltage source connected in series:
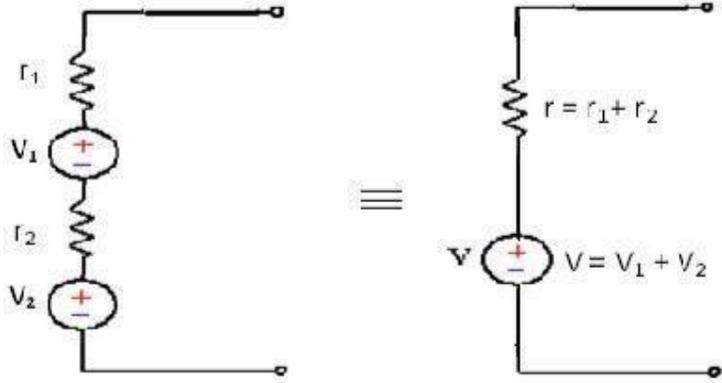

Ideal voltage source connected in parallel:
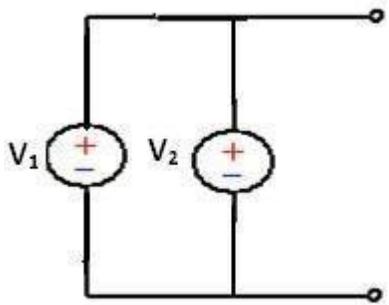
When two ideal voltage sources of emf's V1 & V2 are connected in parallel, what voltage appears across its terminals is ambiguous.

Hence such connections should not be made.

However if V1 = V2= V, then the equivalent voltage some is represented by V.

In that case also, such a connection is unnecessary as only one voltage source serves the purpose.

Practical voltage sources connected in parallel:

Equivalent Circuit   Single Equivalent Voltage Source
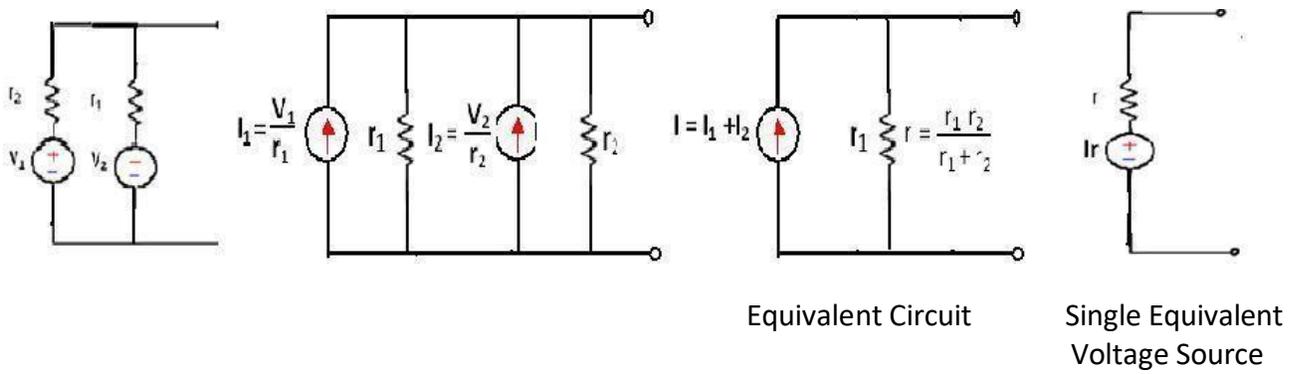
Ideal current sources connected in series:
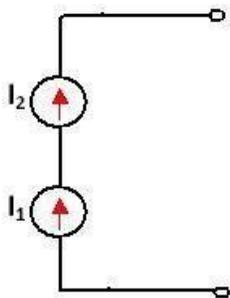
When ideal current sources are connected in series, what current flows through the line is ambiguous. Hence such a connection is not permissible.

However, it I1 = I2 = I, then the current in the line is I.

But, such a connection is not necessary as only one current source serves the purpose.

Practical current sources connected in series:
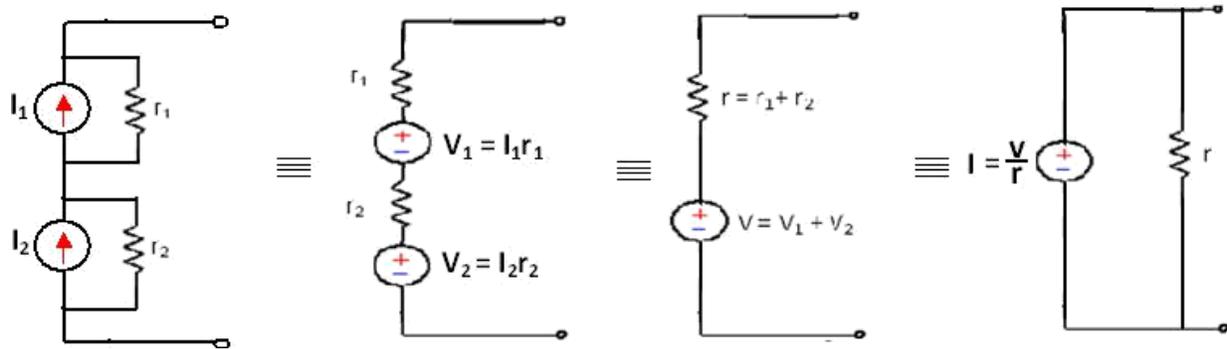
Ideal current sources connected in parallel
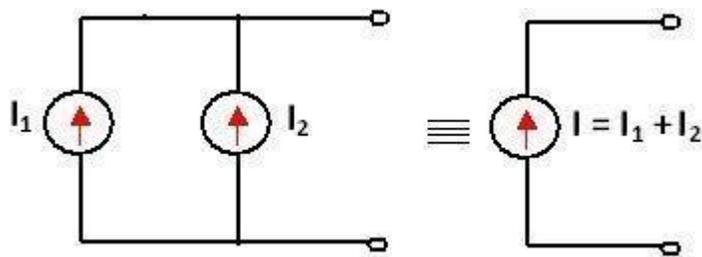
Two ideal current sources in parallel can be replaced by a single equivalent ideal current source.

Practical current sources connected in parallel
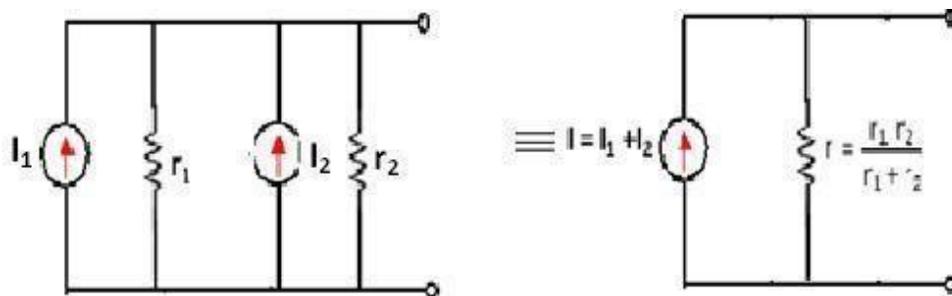
Source transformation:

A current source or a voltage source drives current through its load resistance and the magnitude of the current depends on the value of the load resistance.

Consider a practical voltage source and a practical current source connected to the same load resistance RL as shown in the figure
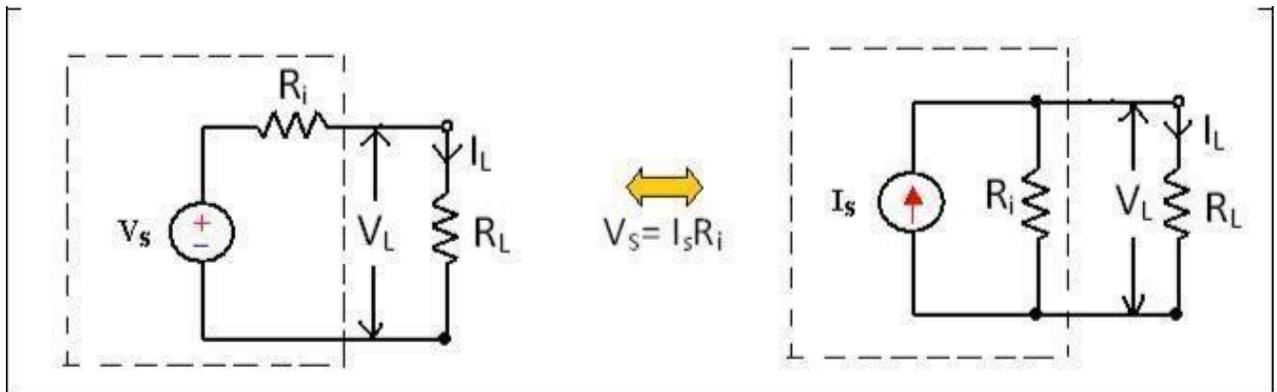
R1's in figure represents the internal resistance of the voltage source VS and current source IS.

Two sources are said to be identical, when they produce identical terminal voltage VL and load current IL.

The circuit in figure represents a practical voltage source & a practical current source respectively, with load connected to both the sources.

The terminal voltage VL and load current IL across their terminals are same.

Hence the practical voltage source & practical current source shown in the dotted box of figure are equal.

The two equivalent sources should also provide the same open circuit voltage & short circuit current.

From fig (a)

$$IL = \frac{Vs}{R+R_L}$$

From fig (b)

$$IL = I\frac{r}{R+R_L}$$

$$\therefore \frac{Vs}{R+R_L} = I$$

$$VS = IR \quad \text{or} \quad I = \frac{Vs}{R}$$

Hence a voltage source Vs in series with its internal resistance R can be converted into a current source $I = \dfrac{Vs}{R}$, with its internal resistance R connected in parallel with it. Similarly a current source I in parallel with its internal resistance R can be converted into a voltage source V = IR in series with its internal resistance R.

R-L-C **Parameters:**

1. **Resistance:**

Resistance is that property of a circuit element which opposes the flow of electric current and in doing so converts electrical energy into heat energy.

It is the proportionality factor in ohm's law relating voltage and current.

Ohm's law states that the voltage drop across a conductor of given length and area of cross section is directly proportional to the current flowing through it.

$$R \propto i$$

$$V = Ri$$

$$i = \frac{v}{R} = GV$$

Where the reciprocal of resistance is called conductance G. The unit of resistance is ohm and the unit of conductance is mho or Siemens.

When current flows through any resistive material, heat is generated by the collision of electrons with other atomic particles. The power absorbed by the resistor is converted to heat and is given by the expression

P= vi= i2R where i is the resistor in amps, and v is the voltage across the resistor in volts.

Energy lost in a resistance in time t is given by

$$W = \int_0^t p\, dt = pt = i^2 Rt = \frac{v2}{R} t$$

2. **Inductance:**

Inductance is the property of a material by virtue of which it opposes any change of magnitude and direction of electric current passing through conductor. A wire of certain length, when twisted into a coil becomes a basic conductor. A change in the magnitude of the current changes the electromagnetic field.

Increase in current expands the field & decrease in current reduces it. A change in current produces change in the electromagnetic field. This induces a voltage across the coil according to Faradays laws of Electromagnetic Induction.

Induced Voltage $V = L\dfrac{di}{dt}$

V = Voltage across inductor in volts

I = Current through inductor in amps

$di = \dfrac{1}{L} v\, dt$

Integrating both sides,

$$\int_0^t di = \frac{1}{L}\int_0^t v\, dt$$

Power absorbed by the inductor $\quad P = VI = Li\,\dfrac{di}{dt}$

Energy stored by the inductor

$$W = \frac{\int_0^t P\, dt}{} = \int_0^t Li\,\frac{di}{dt}\, dt = \frac{Li^2}{2}$$

$$W = \frac{Li^2}{2}$$

Conclusions:

1) $V = L\dfrac{di}{dt}$

      The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.

2) For minute change in current within zero time (dt = 0) gives an infinite voltage across the inductor which is physically not at all feasible. In an inductor, the current cannot change abruptly. An inductor behaves as open circuit just after switching across dc voltage.

3) The inductor can store finite amount of energy, even if the voltage across the inductor is zero.
4) A pure inductor never dissipates energy, it only stores it. Hence it is also called as a non–dissipative passive element. However, physical inductor dissipates power due to internal resistance.

Ex: The current in a 2H inductor raises at a rate of 2A/s .Find the voltage across the inductor the energy stored in the magnetic field at after 2sec.

Sol:

$$V = L\frac{di}{dt}$$

$$= 2 \times 2 = 4V$$

$$W = \frac{1}{2}Li^2 = \frac{1}{2} \times 2 \times (4)^2 = 16\,J$$

3. Capacitance:

1) A capacitor consists of two metallic surfaces or conducting surfaces separated by a dielectric medium.
2) It is a circuit element which is capable of storing electrical energy in its electric field.
3) Capacitance is its capacity to store electrical energy.
4) Capacitance is the proportionality constant relating the charge on the conducting plates to the potential.

Charge on the capacitor $q \propto V$

$$q = CV$$

Where `C` is the capacitance in farads, if q is charge in coulombs and V is the potential difference across the capacitor in volts.

The current flowing in the circuit is rate of flow of charge

$$i = \frac{dq}{dt} = C\frac{dv}{dt} \qquad\qquad \therefore i = C\frac{dv}{dt}$$

The capacitance of a capacitor depends on the dielectric medium & the physical dimensions. For a parallel plate capacitor, the capacitance

$$C = \frac{\epsilon A}{D} = \epsilon_0\,\epsilon_r\frac{A}{D}$$

A is the surface area of plates   D is the separation between plates

€ is the absolute permeability of medium€0 is the absolute permeability of free

space €r is the relative permeability of medium

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

$$\frac{dv}{dt} = \frac{i}{c}$$

$$V = \frac{1}{c}\int i\,dt$$

The power absorbed by the capacitor $P = vi = vc\frac{dv}{dt}$

Energy stored in the capacitor $W = \int_0^t P\,dt = \int_0^t VC\frac{dv}{dt}dt$

$$= C\int_0^t v\,dv = \frac{1}{2}cv^2 \quad \textbf{Joules}$$

This energy is stored in the electric field set up by the voltage across capacitor.

Conclusions:

1. The current in a capacitor is zero, if the voltage across it is constant, that means the capacitor acts as an open circuit to dc
2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible.
   - In a fixed capacitor, the voltage cannot change abruptly
   - A capacitor behaves as short circuit just after switching across dc voltage.
3. The capacitor can store a finite amount of energy, even if the current through it is zero.
4. A pure capacitor never dissipates energy but only stores it hence it is called non-dissipative element.

Kirchhoff`s Laws:

Kirchhoff's laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter.

Kirchhoff`s laws, two in number, are particularly useful in determining the equivalent resistance of a complicated network of conductors and for calculating the currents flowing in the various conductors.

1. Kirchhoff`s Current Law (KCL)

In any electrical network, the algebraic sum of the currents meeting at a point (or junction) is Zero.

That is the total current **entering** a junction is equal to the total current **leaving** that junction.

Consider the case of a network shown in Fig (a*).*

$I_1+(-I_2)+(I_3)+(+I_4)+(-I_5) = 0$

$I_1+I_4-I_2-I_3-I_5 = 0$

Or

$I_1+I_4 = I_2+I_3+I_5$

Or

Incoming currents =Outgoing currents
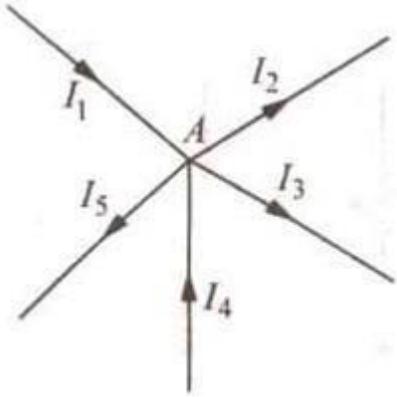
2. **Kirchhoff's Mesh Law or Voltage Law (KVL)**

In any electrical network, the algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.f.'s. in that path is zero.

That is, $\sum IR + \sum e.m.f = 0$ round a mesh

It should be noted that algebraic sum is the sum which takes into account the polarities of the voltage drops.

That is, if we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started.

Hence, it means that all the sources of emf met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

Determination of Voltage Sign

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs.

(a) **Sign of Battery E.M.F.**

A *rise* in voltage should be given a + ve sign and *a fall* in voltage a -ve sign. That is, if we go from the -ve terminal of a battery to its +ve terminal there is a *rise* in potential, hence this voltage should be given a + ve sign.

And on the other hand, we go from +ve terminal to -ve terminal, then there is *a fall* in potential, hence this voltage should be preceded by a -ve sign.

The sign of the battery e.m.f is independent of the direction

E		E		V		V	
A	B	A	B	A +	– B	A –	+ B

| Rise in	Fall in	R	
Voltage	Voltage	motion	motion
+E	–E	Fall in	Rise in
		Voltage	Voltage
		–V = –IR	+ V = + IR
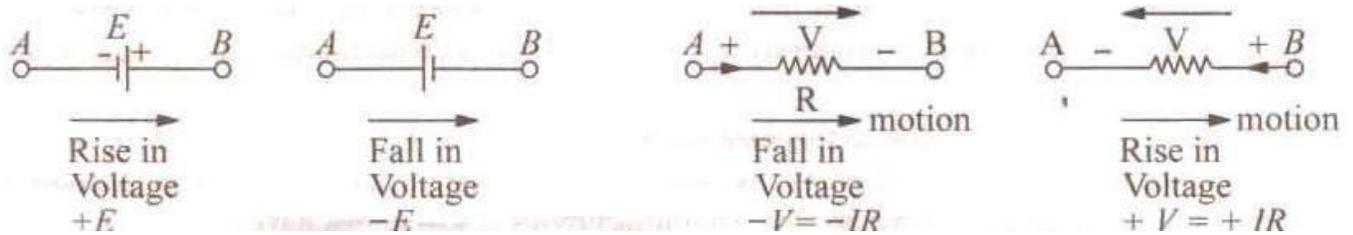
of the current through that branch.

(b) **Sign of *IR* Drop**

Now, take the case of a resistor (Fig. 2.4). If we go through a resistor in the *same* direction as the
current, then there is a fall in potential because current flows from a higher to a lower
potential..
Hence, this voltage fall should be taken -ve. However, if we go in a direction opposite to that of the
current, then there is a *rise* in voltage. Hence, this voltage rise should be given a positive sign.

Consider the closed path *ABCDA* in Fig .
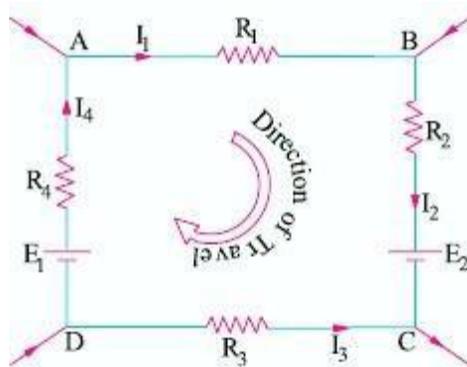
As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs :

I_1R_1 is - ve (fall in potential)
I_2R_2 is - ve (fall in potential)
I_3R_3 is + ve (rise in potential)
I_4R_4 is - ve (fall in potential)
E_2 is - ve   (fall in potential)
E_1 is + ve (rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4 R_4 - E_2 + E_1 = 0$$

Or $\quad I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4 = E_1 - E_2$

Assumed Direction of Current:

In applying Kirchhoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the question, the current will be found to have a minus sign.

If the answer is positive, then assumed direction is the same as actual direction. However, the important point is that once a particular direction has been assumed, the same should be used throughout the solution of the question.

Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents. However, in the case of alternating currents and voltages, any e.m.f. of self-inductance or that existing across a capacitor should be also taken into account.

VOLTAGE-CURRENT RELATIONSHIPS FOR PASSIVE ELEMENTS

The Three Passive Elements are Resistance, Inductance and Capacitance. The behavior of these three elements along with the respective voltage-current relationship is given in the table.

Element	Basic Relation	Voltage across, If Current Known	Current through, If Voltage Known
R	$R = \dfrac{V}{I}$	$V_R(t) = R\, i_R(t)$	$i_R(t) = \dfrac{V_R(t)}{R}$
L	$L = \dfrac{N\emptyset}{I}$	$V_L(t) = L\dfrac{di(t)}{dt}$	$i_L(t) = \dfrac{1}{L}\int_{-\infty}^{t} V(t)dt$
C	$C = \dfrac{Q}{V}$	$V_C(t) = \dfrac{1}{C}\int_{-\infty}^{t} i_C(t)dt$	$i_C(t) = C\dfrac{dv(t)}{dt}$

Table

1. A current waveform flowing through an inductor of 1mH is shown in the figure. Obtain and sketch the waveform of voltage across the inductor.
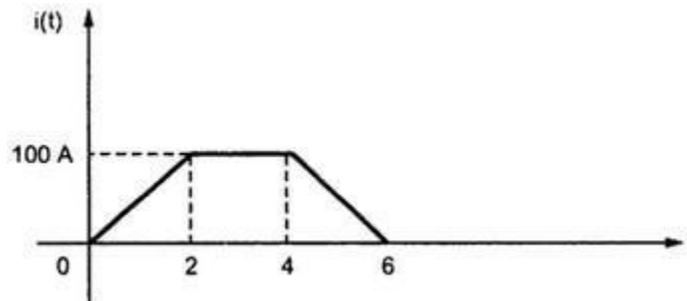
Solution:

From the given waveform,

For 0<t<2, i(t) is a straight line of slope=(100/2)=50

Therefore, i(t)=50t and $\dfrac{di(t)}{dt}$=50

For 2<t<4, i(t)=100 and $\dfrac{di(t)}{dt}$=0

For 4<t<6, i(t) is a straight line of slope=-(100/2)=-50

Therefore, i(t)=-50t and $\dfrac{di(t)}{dt}$=-50

Now, $V_L(t) = L \dfrac{di(t)}{dt}$

$\qquad\qquad =1*10^{-3}*50=0.05V \qquad\qquad\qquad 0<t<2$

$\qquad\qquad =1*10^{-3}*0=0V \qquad\qquad\qquad 2<t<4$

$\qquad\qquad =1*10^{-3}*(-50)=-0.05V \qquad\qquad 4<t<6$

The voltage waveform is shown in following figure.
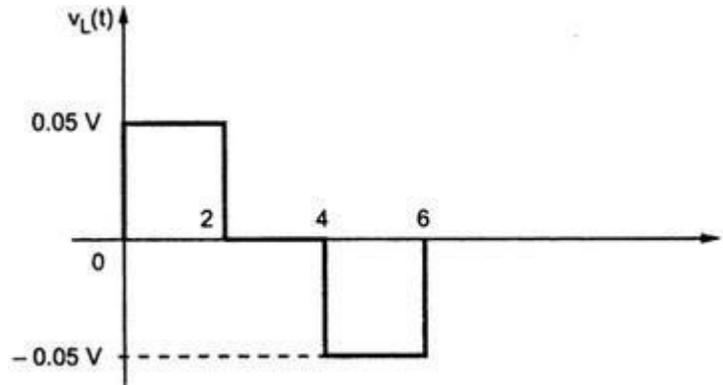
2. A 0.5uF capacitor has voltage waveform v(t) as shown in following figure, plot i(t) as function of t ?
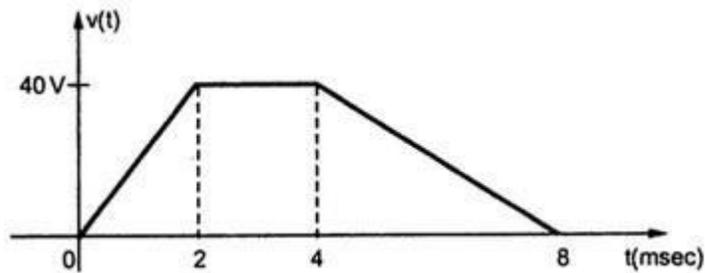
Solution:

From the given waveform,

For 0<t<2, v(t) is a ramp of slope =(40/2)=20

Therefore   v(t)=20t

Therefore $i(t)=C\dfrac{dv(t)}{dt}=0.5*10^{-6}*20=1*10^{-5}A=10uA$

For 2<t<4, v(t) is constant

Therefore   v(t)=40V

Therefore $i(t)=C\dfrac{dv(t)}{dt}=0.5*10^{-6}*0=0A$

For 4<t<8, v(t) is a ramp with slope = $\dfrac{0-40}{8-4}=-10$

Therefore   v(t)=-10t+80   (According to straight line equation i.e. y=mx+c)

Therefore $i(t)=C\dfrac{dv(t)}{dt}=0.5*10^{-6}*(-10)=-5uA$

The current waveform is shown in following figure
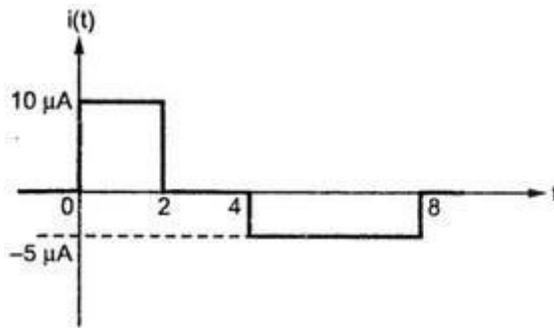
3. A Pure Inductance Of 3mh Carries A Current Of The Waveform Shown In Fig. Sketch The Waveform Of V (t) And P(t).Determine The Average Value Of Power
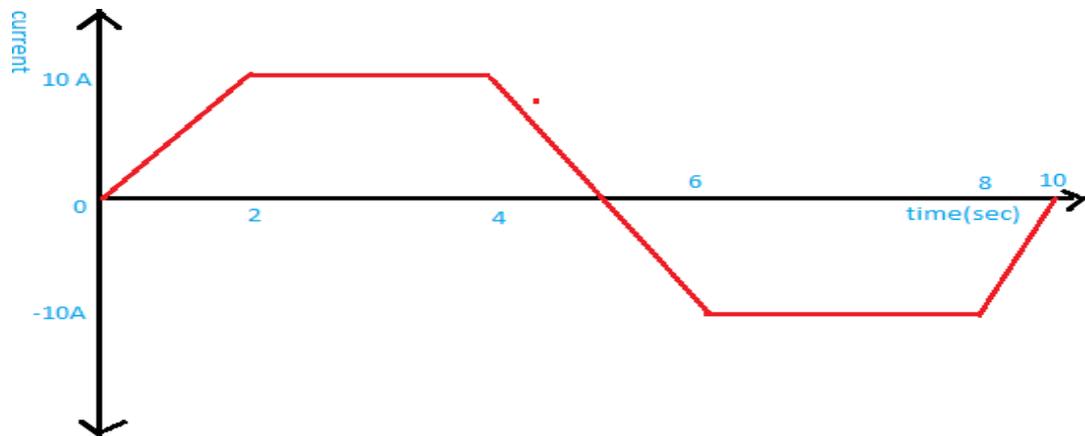
Fig

Solution:

i (t)=5t for 0<t<2

i (t)=10 for 2<t<4

i (t)=-10t+50 for 4<t<6

i (t)=-10 for 6<t<8

$i(t)=5t-50$ for $8<t<10$

For $0<t<2$, $V_L(t)=L\frac{di(t)}{dt}$

$=3*10^{-3}\frac{d(5t)}{dt}$

$=15*10^{-3}v$

For $2<t<4$, $V_L(t)=L\frac{di(t)}{dt}$

$=3*10^{-3}\frac{d(10)}{dt}$

$=0v$

For $4<t<6$, $V_L(t)=L\frac{di(t)}{dt}$

$=3*10^{-3}\frac{d(-10t+50)}{dt}$

$=-30*10^{-3}v$

For $6<t<8$, $V_L(t)=L\frac{di(t)}{dt}$

$=3*10^{-3}\frac{d(-10)}{dt}$

$=0v$

For $8<t<10$, $V_L(t)=L\frac{di(t)}{dt}$

$=3*10^{-3}\frac{d(5t-50)}{dt}$

$=15*10^{-3}v$

The sketch of v(t) is shown in fig.
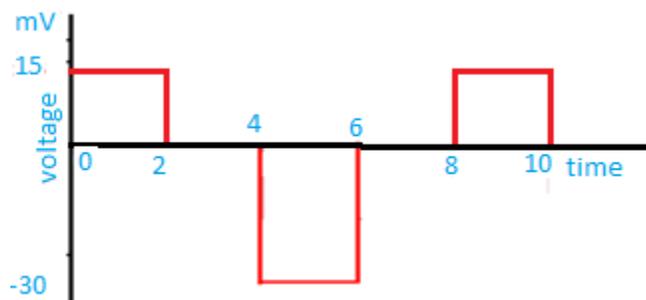
Fig.

For 0<t<2,   P(t)=v(t)i(t)

=75*10-3t W

For 2<t<4,   P(t)=v(t)i(t)

=0W

For 4<t<6,   P(t)=v(t)i(t)

=-30*(-10t+50)*10^{-3}=-0.3W (at t=4)

=0W (at t=5)

=0.3W (at t=6)

For 6<t<8,   P(t)=v(t)i(t)

=0W

For 8<t<10,   P(t)=v(t)i(t)

=15*(5t-50)*10^{-3}=-0.15W (at t=8)

=-0.075W (at t=9)

=0W (at t=10)
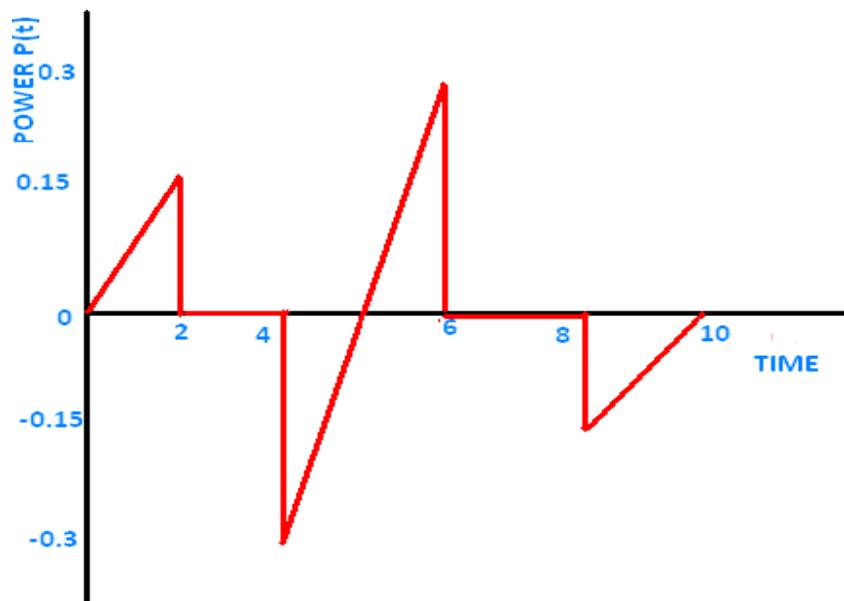
4. Draw the waveforms for current, power for the following elements if a voltage input shown in figure is applied to these elements.

      i.   R=1 OHM
      ii.   L=1H
      iii.   C=1F
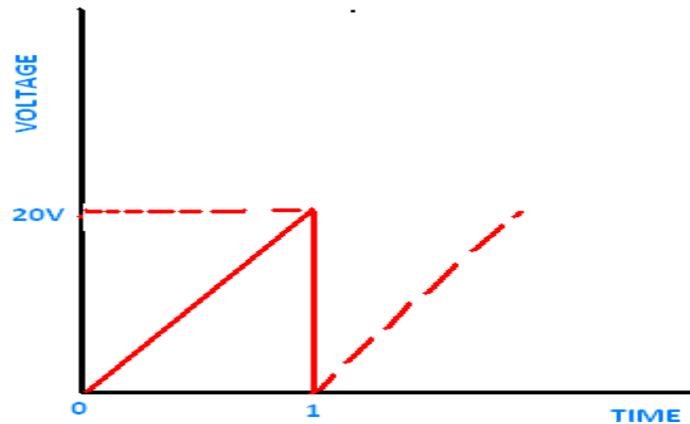
SOLUTION:

From the figure, v (t) is a straight line with slope = $\frac{20-0}{1-0}$=20, For 0<t<1

Therefore v (t) =20t

i.   **R=1 OHM**

    The voltage and current relation of a resistor is given by, v (t) =R i(t)

                                   i(t) =20t/1=20t

    Hence,

    When t=0, i(t)=0A

    When t=0.5, i(t)=10A

    When t=1, i(t)=20A

    Therefore the current waveform for the above values of t and i(t) is shown in figure below
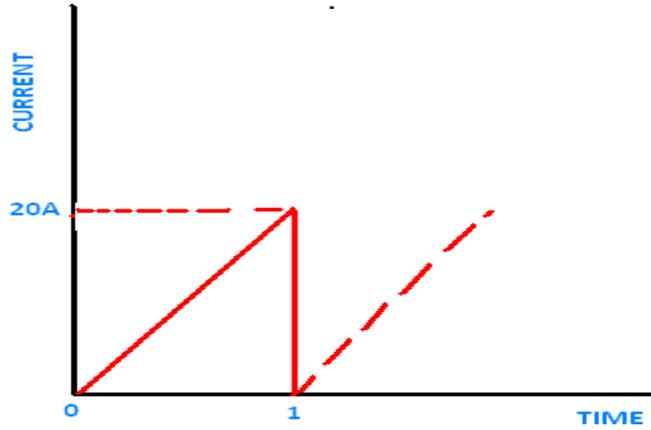
Power, p(t)=v(t)i(t)

    =20t*20t=400t² W

Hence,

When t=0, p(t)=v(t)i(t)=0W

When t=0.4, p(t)=v(t)i(t)=64W

When t=0.8, p(t)=v(t)i(t)=256W

When t=, p(t)=v(t)i(t)=400W

Therefore the power wave form for the above values of t and p(t) is shown in below figure.
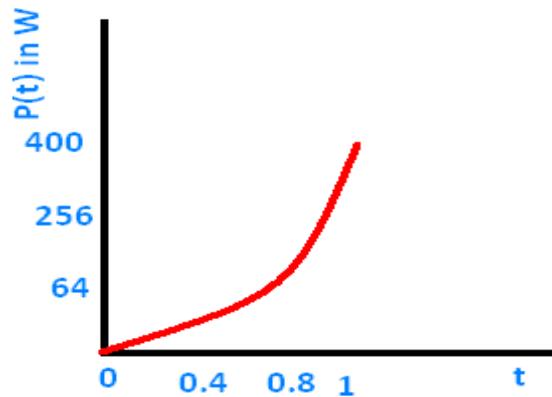
ii.   <u>**L=1 H**</u>

The voltage and current relation of a inductor is given by,

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} V(t)dt$$

$$i(t) = \frac{1}{1} \left[\int_{-\infty}^{0} V(t)dt + \int_{0}^{t} V(t)dt \right]$$

$$i(t)=0+\int_0^t V(t)dt=\int_0^t 20tdt=10t^2$$

Therefore the current waveform is shown in below figure.
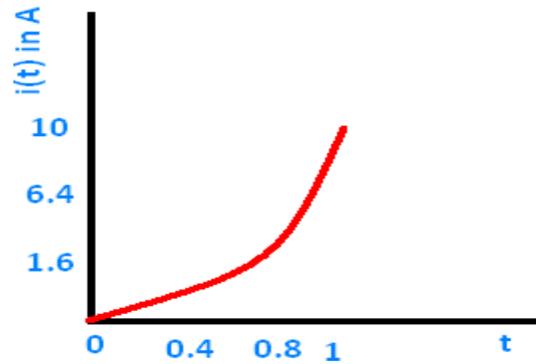
Power, p(t)=v(t)i(t)=20t*10t²=200t³ W

Therefore the power waveform is shown in below figure
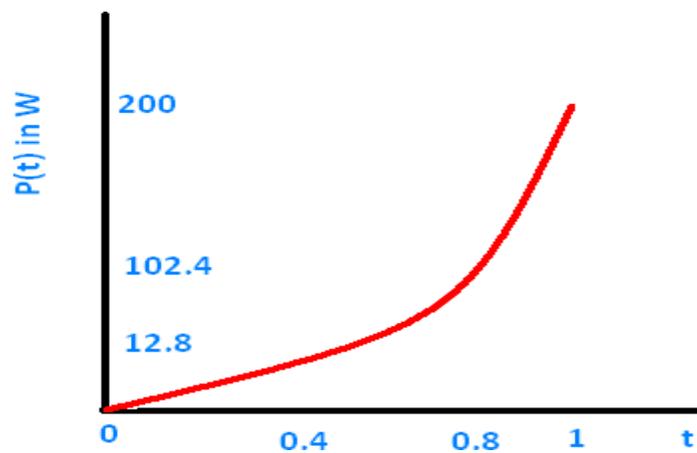
iii.   <u>**C=1 F**</u>

The voltage and current relation of a inductor is given by

$$i(t)=C\frac{dV(t)}{dt}$$

$$i(t)=1*\frac{d(20t)}{dt}=20A$$
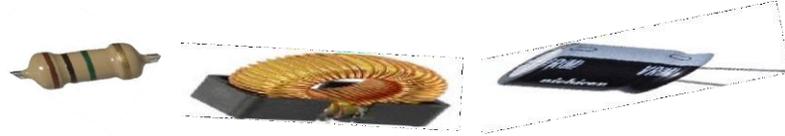
Therefore the current waveform is shown in below figure
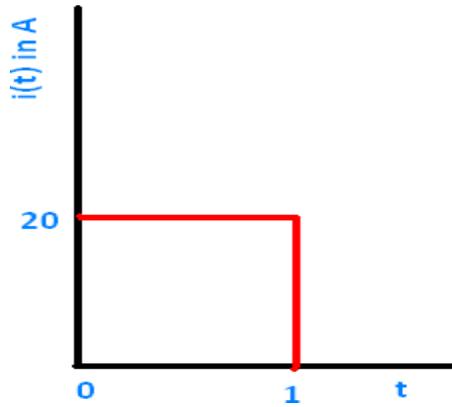
Power, p(t)=v(t)i(t)=20t*20=400t W

Therefore the power waveform is shown in below figure.
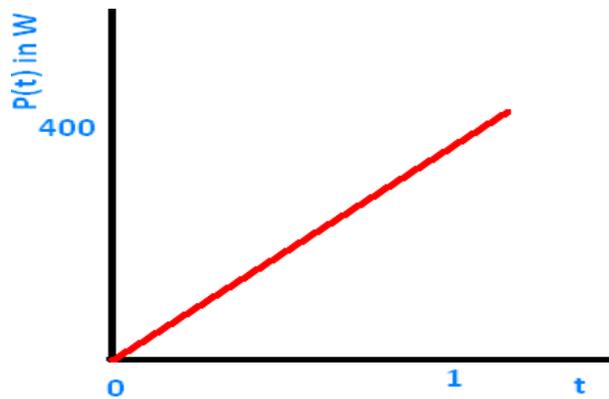
Introduction:

A network is a collection of interconnected electrical components. In general, the electrical networks are made to exchange the energy between different elements .These electrical networks can be constructed either by using Resistors or Inductors or Capacitors or combination of these elements. Network analysis is the process of finding the voltage response or the current response for any element in the network by using the available techniques.

Network Reduction Techniques:

Series Connection of Resistors:

Two or more resistors in a circuit are said to be in series when the current flowing through all the resistors is the same.

fig(a)   fig(b)
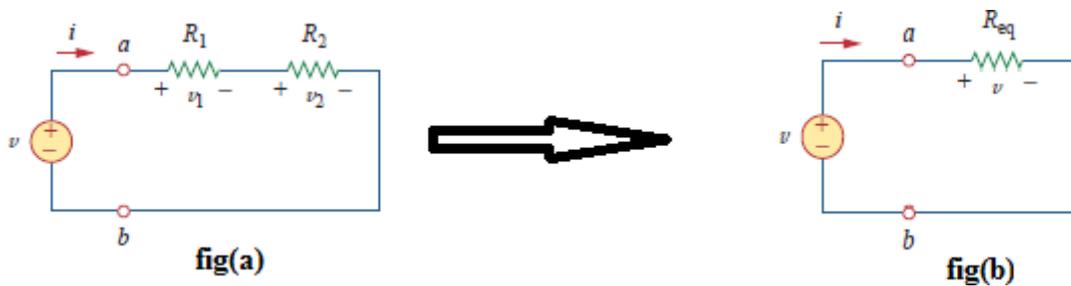
Consider the circuit in fig(a) ,where two resistors R_1 and R_2 are in series, since the same current i flows in both of them. Applying Ohm's law to each of the resistors, we obtain

$v_1 = iR_1$,.......(1) $v_2 = iR_2$...........(2)

If we apply KVL to the loop fig(b) , we have

$-v + v_1 + v_2 = 0$......................... (3)

$v = v1 + v2 = i(R1 + R2)$

$i = v/(R1 + R2)$............................(4)

$v = i\ Req$.............. (5)

Implying that the two resistors can be replaced by an equivalent resistor; that is,

$$Req = R1 + R2$$

Note: The equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

If "n"resistors are in series,   Req=R1+R2+..... Rn

If "n"resistors of same value are in series,   Req=nR

Parallel Connection of Resistors:

Two or more resistors in a circuit are said to be in Parallel when all the resistors are connected to the same nodes and the same voltage is appearing across all these elements.

fig(a)   fig(b)
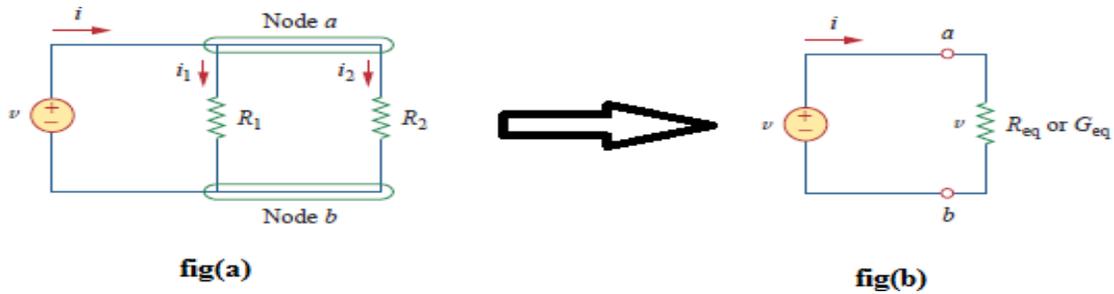
Consider the circuit in fig(a), where two resistors are connected in parallel and therefore have the same voltage across them. From Ohm's law,

$$v = i_1R_1 = i_2R_2 \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$i1 = v/R1, \quad i2 = v/R2 \ldots\ldots\ldots\ldots(2)$

Applying KCL at node a gives the total current i as

$i = i1 + i2 \ldots\ldots\ldots\ldots(3)$

Substituting Eq. (2) into Eq. (3), we get

$i = v/R_1 + v/R_2 = v(1/R_1 + 1/R_2) = v/Req$

$1/Req = 1/R_1 + 1/R_2$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Note: The equivalent resistance of two parallel resistors is equal to the product   of their resistances divided by their sum.

If "n" resistors are in parallel, $1/Req = 1/R1 + 1/R2 + \ldots\ldots 1/Rn$

If "n" resistors of same value are in parallel, $Req = R/n$

It is often more convenient to use conductance rather than resistance when dealing with resistors in parallel. The equivalent conductance for N resistors in parallel is

$$G_{eq} = G_1 + G_2 + G_3 + \cdots + G_N$$

where $G_{eq} = 1/R_{eq}$, $G_1 = 1/R_1$, $G_2 = 1/R_2$, $G_3 = 1/R_3$, \ldots, $G_N = 1/R_N$.

Note: The equivalent conductance of resistors connected in parallel is the sum of their individual conductances.

Example Problems:

1) Find the Req for the circuit shown in below figure.
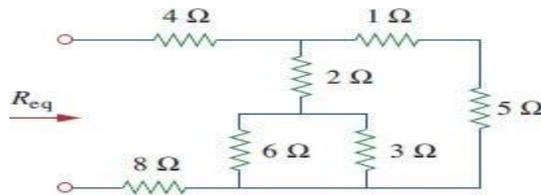
fig(a)

Solution:
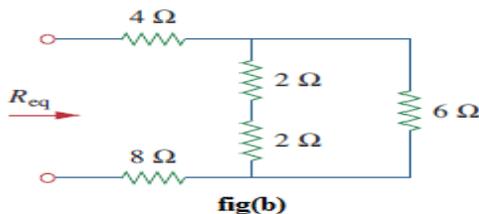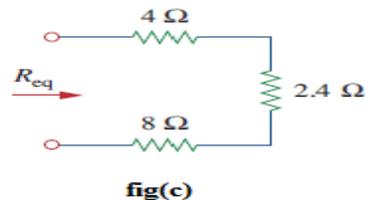
fig(b)   **fig(c)**

To get Req we combine resistors in series and in parallel. The 6 ohms and 3 ohms resistors are in parallel, so their equivalent resistance is

$$6 \, \Omega \, \| \, 3\Omega = \frac{6 \times 3}{6 + 3} = 2 \, \Omega$$

Also, the 1 ohm and 5ohms resistors are in series; hence their equivalent resistance is

$$1 \, \Omega + 5 \, \Omega = 6 \, \Omega$$

Thus the circuit in Fig.(b) is reduced to that in Fig. (c). In Fig. (b), we notice that the two 2 ohms resistors are in series, so the equivalent resistance is

$$2 \, \Omega + 2 \, \Omega = 4 \, \Omega$$

This 4 ohms resistor is now in parallel with the 6 ohms resistor in Fig.(b); their equivalent resistance is

$$4 \, \Omega \, \| \, 6 \, \Omega = \frac{4 \times 6}{4 + 6} = 2.4 \, \Omega$$

The circuit in Fig.(b) is now replaced with that in Fig.(c). In Fig.(c), the three resistors are in series. Hence, the equivalent resistance for the circuit is

$$R_{eq} = 4 \, \Omega + 2.4 \, \Omega + 8 \, \Omega = 14.4 \, \Omega$$

2) Find the Req for the circuit shown in below figure.


fig(a)
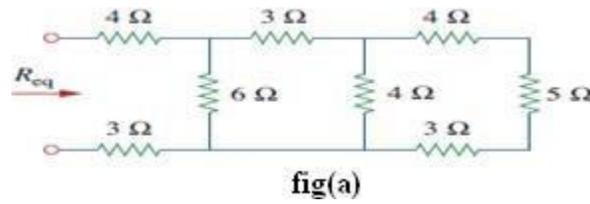
Solution:

In the given network 4 ohms, 5 ohms and 3 ohms comes in series then equivalent resistance is 4+5 + 3 = 12 ohms
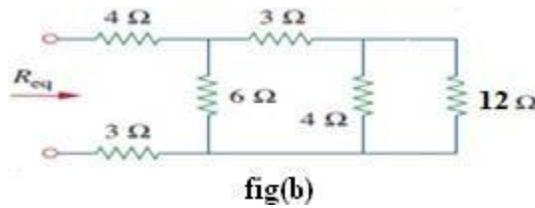

fig(b)

From fig(b), 4 ohms and 12 ohms are in parallel, equivalent is 3 ohms
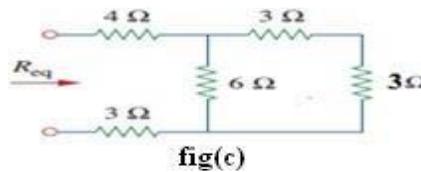
fig(c)

From fig(c), 3 ohms and 3 ohms are in series, equivalent resistance is 6 ohms
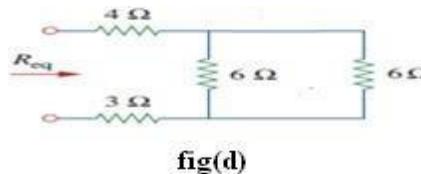

fig(d)

From fig(d), 6 ohms and 6 ohms are in parallel, equivalent resistance is 3 ohms
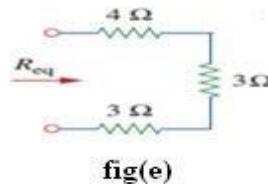
fig(e)

From fig(e), 4 ohms, 3 ohms and 3 ohms are in series .Hence Req = 4+ 3+ 3 =10 ohms

Series Connection of Inductors:

Two or more inductors in a circuit are said to be in series when the current flowing through all the inductors is the same.

fig(a)   fig(b)
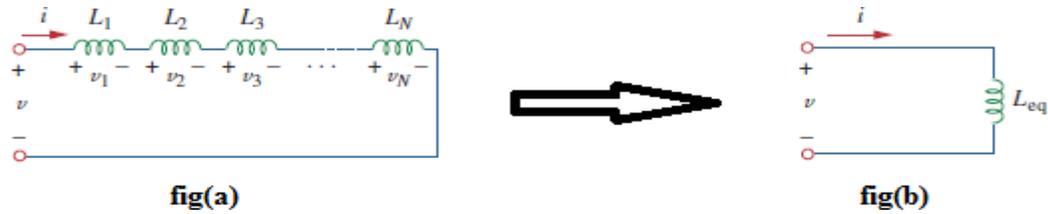
Consider a series connection of N inductors, as shown in Fig(a),with the equivalent circuit shown in Fig(b). The inductors have the same current through them. Applying KVL to the loop,

$V = V1 + V2 + \ldots\ldots + Vn = L1 \cdot di/dt + L2 \cdot di/dt + \ldots\ldots Ln \cdot di/dt$

$= di/dt \ (L1 + L2 + \ldots + Ln)$

$= di/dt(L \ eq)$

where Leq is the equivalent Inductance of all the Inductances L1 ,L2 ,….and Ln in series.

Hence Leq of a series circuit consisting of n Inductances L1 ,L2, … Ln connected in series is given by

$$Leq = L1 + L2 + \ldots + Ln$$

Note: The equivalent Inductance Leq of a circuit with n Inductances L1, L2, ….Ln connected in series is equal to the sum of the individual Inductances L1 ,L2, … Ln

Parallel Connection of Inductors:

fig(b)
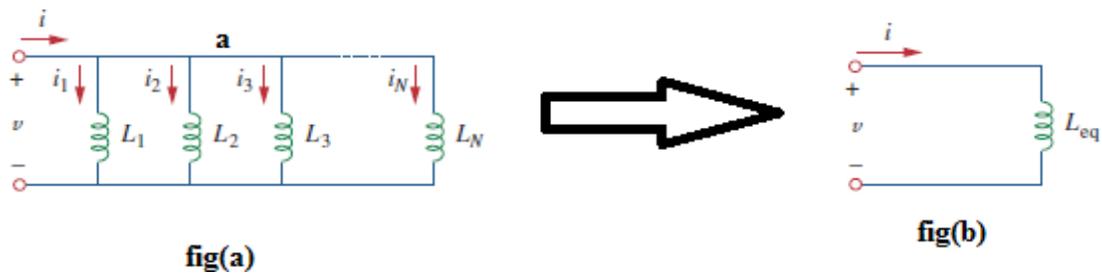
fig(a)

Two or more inductors in a circuit are said to be in Parallel when all the circuit inductors are connected to the same nodes and the same voltage is appearing across all these inductors

Consider the circuit in fig(a) and using the Kirchhoff's Current Law (KCL) at the node 'a' the governing equation can be written as:

$$i = i1 + i2 + \ldots\ldots + in \ldots\ldots\ldots (1)$$

In terms of the applied voltage V , the individual Inductances L1 ,L2, … Ln the above equation can be written as:

$i = (1/L1)\int vdt + (1/L2)\int vdt + \dots + (1/Ln)\int vdt = [(1/L1) + (1/L2) + \dots + (1/Ln)]\int vdt$ ...... (2)

Similarly for the circuit in fig(b) we can write the governing equation as:

$i = (1/Leq)\int v$ ....(3)

where Leq is the equivalent Inductance of all the Inductances L1 , L2 ,….and Ln in parallel.
Since current is the same in the above two equations we find that

$1/Leq = (1/L1 + 1/L2 + \dots 1/Ln)$

Hence Leq of a parallel circuit consisting of n Inductances L1 , L2 ,….and Ln connected in parallel is given by :

$$1/Leq = (1/L1 + 1/L2 + \dots 1/Ln)$$

Note: The reciprocal of the equivalent inductance is the sum of the reciprocals of the inductances.

Problem: Determine the equivalent inductance in the given network.
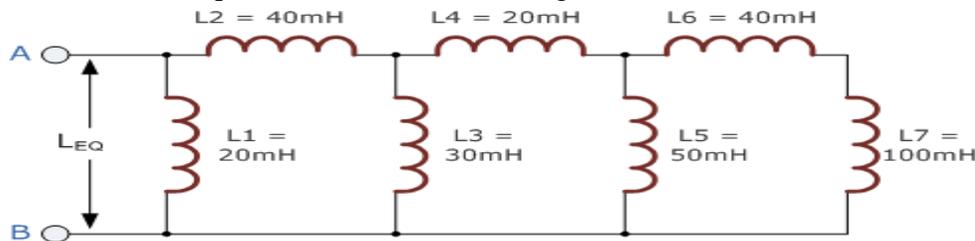
Solution:
Calculating the first inductor branch L_A, where an Inductor L_5 in parallel with inductors L_6 and L_7.

$$LA = \frac{L5 \times (L6 + L7)}{L5 + L6 + L7} = \frac{50mH \times (40mH + 100mH)}{50mH + 40mH + 100mH} = 36.8mH$$

Calculating the second inductor branch L_B, where Inductor L_3 in parallel with inductors L_4 and L_A.

$$LB = \frac{L3 \times (L4 + LA)}{L3 + L + LA} = \frac{30mH \times (20mH + 36.8mH)}{30mH + 20mH + 36.8mH} = 19.6mH$$

Calculate the equivalent circuit inductance LEQ, where Inductor L_1 in parallel with inductors L_2 and L_B.

$$LEQ, = \frac{L1 \times (L2+LB)}{L1+L2+LB} = \frac{20mH \times (40mH+19.6mH)}{20mH+40mH+19.6mH} = 15mH$$

Series Connection of Capacitors:
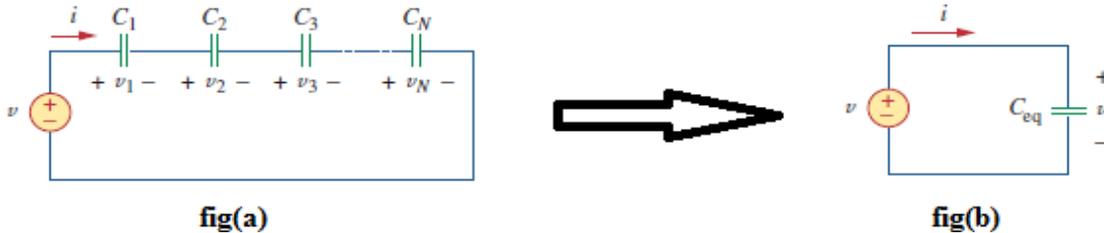
fig(a)   **fig(b)**

Two or more capacitors in a circuit are said to be in series when the current flowing through all the capacitors is the same.

Fig (a) above shows n capacitances C1 ,C2, … Cn connected in series along with a Voltage source V and V1 , V2 …… Vn are the voltage drops across the capacitances C1 , C2 ,….and Cn. Fig (b) shows the same circuit with it's equivalent Capacitance Ceq.

Applying KVL to the circuit in figure (a) we can write: v = v1 + v2 + v3 + ….+ vn ....(1)
V = (1/C1) $\int$ idt +(1/C2) $\int$ idt + … +(1/Cn) $\int$ idt = [(1/C1) +(1/C2) + … +(1/Cn)] $\int$ idt........ (2)

Similarly for the circuit in fig.(b) we can write the governing equation as:
v = (1/Ceq) $\int$idt ....(3)
From the above two equations i.e eq(2) and eq (3) for 'v' we get
1/Ceq = (1/C1) +(1/C2) + … +(1/Cn)
Hence Ceq of a circuit consisting of n capacitances C1 , CL2 ,….and Cn connected in series is given by :

1/Ceq = (1/C1) +(1/C2) + … +(1/Cn)

Note: The reciprocal of the equivalent capacitance is the sum of the reciprocals of the inductances.

Parallel Connection of Capacitors:

Two or more capacitors in a circuit are said to be in Parallel when all the capacitors are connected to the same nodes and the same voltage is appearing across all these capacitors

fig(a)   fig(2)
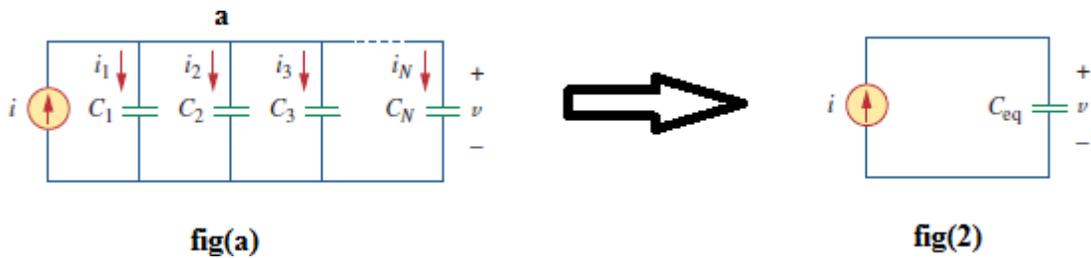
Applying KCL to the circuit in figure (a) we can write: $i = i1 + i2 + i3 + \ldots + in$ ...... (1)

$i = C1\,(dv/dt) + C2\,(dv/dt) + \ldots + Cn\,dv/dt) = (C1 + C2 + \ldots + Cn)\,(dv/dt)$............(2)

Similarly for the circuit in figure (b) we can write the governing equation as:

$i = Ceq\,(dv/dt)$............(3)

From the eq(2) and eq(3),we get

Ceq=C1+C2+... +Cn

Note: The equivalent capacitance of a circuit consisting of n capacitances C1 , CL2 ,....and Cn connected in parallel is the sum of the individual capacitances.

Problem: Determine the equivalent capacitance between AB in the given circuit.
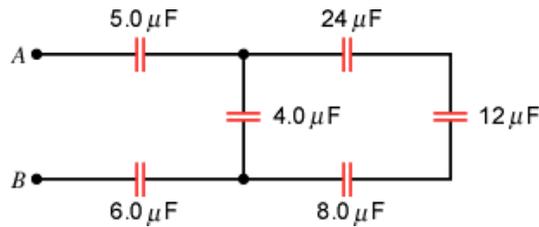
Solution: In the given circuit 24uF,12uF and 8uF are in series. The equivalent capacitance of this three capacitors is

$$\frac{1}{C_s} = \frac{1}{24\ \mu F} + \frac{1}{12\ \mu F} + \frac{1}{8.0\ \mu F} \qquad \text{or} \qquad C_s = 4.0\ \mu F$$

This 4.0μF capacitance is in parallel with the 4.0μF capacitance. Then the equivalent capacitance is

$$C_p = 4.0\ \mu F + 4.0\ \mu F = 8.0\ \mu F$$

This 8.0μF,5.0uF and 6.0uF are in series .Hence , the overall equivalent capacitance is

$$\frac{1}{C_s} = \frac{1}{5.0\ \mu F} + \frac{1}{8.0\ \mu F} + \frac{1}{6.0\ \mu F} \qquad \text{or} \qquad C_s = \boxed{2.0\ \mu F}$$

Star to Delta and Delta to Star Transformations:

Like in series and parallel connections, electrical components may be connected in Star or Delta configurations as shown in the figure below (with Resistances). Many a times circuits have to be transformed from Star to equivalent Delta and Delta to equivalent Star configurations such that the net terminal Resistances (Impedances)across the terminals are the same. We will show this transformation methodology and the resulting configurations for both Delta to Star and Star to Delta one by one.

(a) Star connection   (b) Delta connection
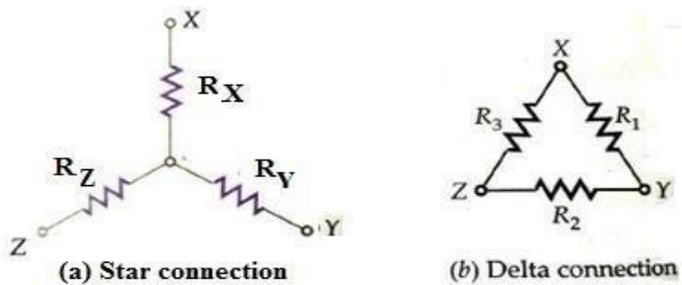
Delta to Star Transformation:

The circuit configurations are identical provided the net resistances across the terminal pairs XY, YZ and ZX in both connections are the same. In Star Connection they are:

$R_{X\text{-}Y}$ = $R_X + R_Y$...(1)

$R_{Y\text{-}Z}$ = $R_Y + R_Z$...(2)

$R_{Z\text{-}X}$ = $R_Z + R_X$...(3)

Similarly in Delta connection they are:

$R_{X\text{-}Y}$ = R1//(R2+R3) = $\dfrac{R1(R2+R3)}{R1+R2+R3}$ ....(4)

$R_{Y\text{-}Z}$ = R2//(R1+R3) = $\dfrac{R2(R1+R3)}{R1+R2+R3}$ ....(5)

$R_{Z\text{-}X}$ = R3//(R1+R2) = $\dfrac{R3(R1+R2)}{R1+R2+R3}$ ....(6)

By equating the respective equations, we get

$$R_X + R_Y = \frac{R1(R2+R3)}{R1+R2+R3} \quad \ldots\ldots\ldots\ldots(7)$$

$$R_Y + R_Z = \frac{R2(R1+R3)}{R1+R2+R3} \quad \ldots\ldots\ldots\ldots(8)$$

$$R_Z + R_X = \frac{R3(R1+R2)}{R1+R2+R3} \quad \ldots\ldots\ldots\ldots(9)$$

By subtracting equation 8 from equation 7 given above, we get

$$RX - RZ = \frac{R1R2 + R1R3}{R1 + R2 + R3} - \frac{R2R1 + R2R3}{R1 + R2 + R3} \ldots\ldots\ldots\ldots(10)$$

Then adding this equation to equation 9 above i.e. (R_Z+R_X) we get:

$$2RX = \frac{R1R2 + R1R3 - R2R1 - R2R3 + R3R1 + R3R2}{R1 + R2 + R3}$$

$$= \frac{2R1R3}{R1 + R2 + R3}$$

$$RX = \frac{R1R3}{R1 + R2 + R3}$$

And in a similar way we can get:

$$RY = \frac{R1R2}{R1 + R2 + R3}$$

$$RZ = \frac{R2R3}{R1 + R2 + R3}$$

Where RX, RY and RZ are the equivalent resistances in the Star connection corresponding to the Delta connection with resistances R1, R2 and R3.

Star to Delta Transformation:
Now we have to get the equivalent values of R1, R2 and R3 in Delta connection in terms of the three resistances R_X,R_Y and R_Z in Star connection.
Let us use the equations we got earlier i.e. R_X,R_Y and R_Z in terms of R_1,R_2 and R_3 and get the sum of the three product pairs i.e. $R_X R_Y + R_Y R_Z + R_Z R_X$ as :

$$RXRY + RYRZ + RZRX = \frac{R1^2R2R3 + R2^2R1R3 + R3^2R1R2}{(R1 + R2 + R3)^2}$$

Now let us divide this equation by R_X to get:

$$RY + RZ + \frac{RYRZ}{RX} = \frac{R1R2R3(R1 + R2 + R3)}{RX(R1 + R2 + R3)^2}$$
$$= \frac{R1R2R3}{RX(R1 + R2 + R3)}$$

Now substituting the value of R_X = (R1+R2+R3) / R1.R3 from the earlier equations into the above equation we get:

$$RY + RZ + \frac{RYRZ}{RX} = \frac{R1R2R3}{(R1 + R2 + R3)} \times \frac{(R1 + R2 + R3)}{R1R3} = R2$$

Then similarly dividing the same equation by RY and RZ we get the other two relations as:

$$RX + RZ + \frac{RXRZ}{RY} = R3$$

$$RY + RX + \frac{RXRY}{RZ} = R1$$

Thus we get the three equivalent resistances R1, R2 and R3 in Delta connection in terms of the three resistances RX, RY and RZ in Star connection as :

$$RY + RX + \frac{RXRY}{RZ} = R1$$

$$RY + RZ + \frac{RYRZ}{RX} = R2$$

$$RX + RZ + \frac{RXRZ}{RY} = R3$$

Example problems:

1) Convert the Delta network in a) Fig.(a) to an equivalent star network

(a)   (b)
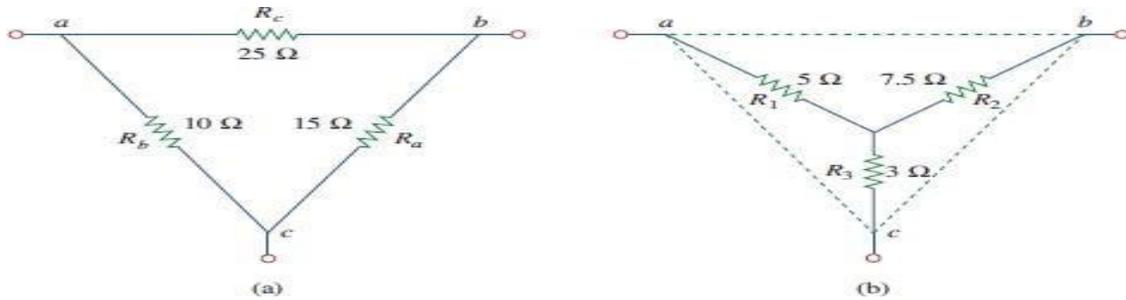
Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\,\Omega$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\,\Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\,\Omega$$

2) Convert the star network in fig(a) to delta network

fig(a)   fig(b)

Solution: The equivalent delta for the given star is shown in fig(b), where

$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = 10 \ \Omega$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = 15 \ \Omega$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833$$

$$= 5 \ \Omega$$

3) Determine the total current I in the given circuit.
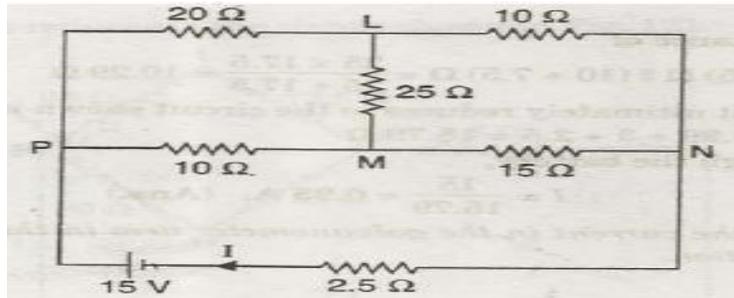
 Solution: Delta connected resistors 25 ohms, 10 ohms and 15 ohms are converted in to star as shown in given figure.

$R_1 = R_{12} R_{31} / R_{12} + R_{23} + R_{31} = 10 \ x \ 25 \ / \ 10 + 15 + 25 = 5$ ohms
$R_2 = R_{23} R_{12} / R_{12} + R_{23} + R_{31} = 15 \ x \ 10 \ / \ 10 + 15 + 25 = 3$ ohms
$R_3 = R_{31} R_{23} / R_{12} + R_{23} + R_{31} = 25 \ x \ 15 \ / \ 10 + 15 + 25 = 7.5$ ohms
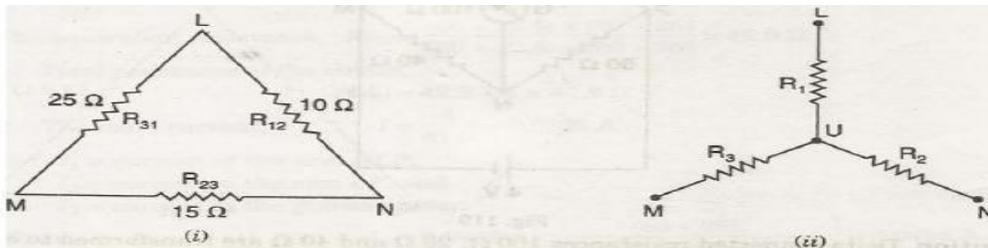
The given circuit thus reduces to the circuit shown in below fig.
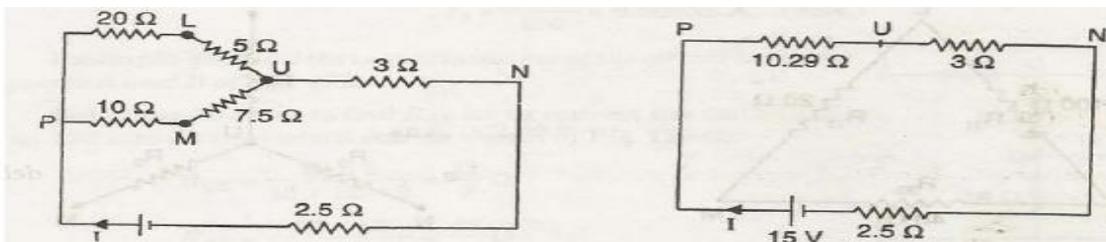
The equivalent resistance of

(20 + 5) ohms || (10 + 7.5) ohms = 25 x 17.5 / 25 + 17.5 = 10.29 ohms

Total resistance = 10.29 + 3 + 2.5 = 15.79 ohms

Hence the total current through the battery,

I = 15 / 15.79 = 0.95 A

**Introduction to Mesh Analysis and Nodal Analysis:**

Mesh Analysis and Nodal Analysis are two important techniques used in network analysis to find out different branch currents and Node voltages. The suitability of each analysis depends mainly on the number of voltage/current sources in the given network. If the voltage sources are more Mesh analysis is suitable and if current sources are more Nodal analysis is more suitable.

Mesh Analysis:

Mesh analysis provides general procedure for analyzing circuits using mesh currents as the circuit variables. Mesh Analysis is applicable only for planar networks. It is preferably useful for the circuits that have many loops .This analysis is done by using KVL and Ohm's law.

Planar circuit: A planar circuit is one that can be drawn in a plane with no branches crossing one another. In the figure below (a) is a planar circuit.

Non-Planar circuit: A planar circuit is one that cannot be drawn in a plane without the branches crossing one another. In the figure below   (b) is a n.on-planar circuit and (c) is a planar circuit but appears like a non-planar circuit


(a)   (b)   (c)

:
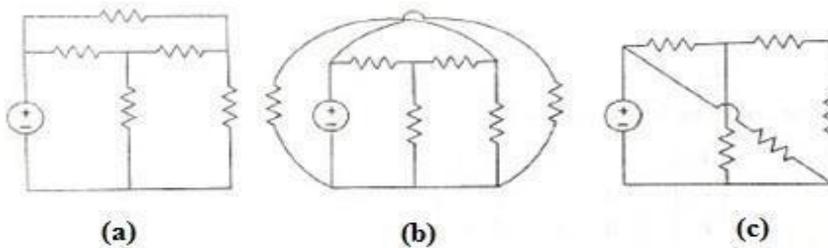
Loop: It is a closed path along the circuit elements.

Mesh: Mesh is a loop which does not contains any loop within it.

Mesh analysis with example:
Determination of mesh currents:
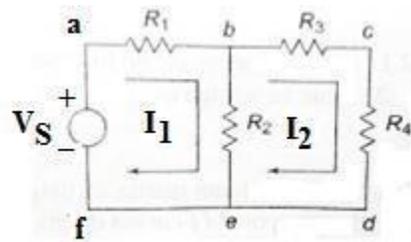
Solution:

Step (1): Identify the no. of meshes in the given circuit.

There are two meshes..Mesh (1)..... **abef** and

Mesh (2) .... **bcde**

Step (2): Apply the KVL to the all meshes.

For mesh (1) by applying KVL... **Vs -I1 .R1 + (I1- I2). R2 =0. ....(1)**

For mesh (2) by applying KVL.... **I2 .(R3)+ I2 (R4)+ (I2- I1). R2=0. ......(2)**

Step (3): solve the above equations for mesh currents.

Problem: Write down the mesh current equations for the circuit shown in the figure below and determine the currents I1 and I2.
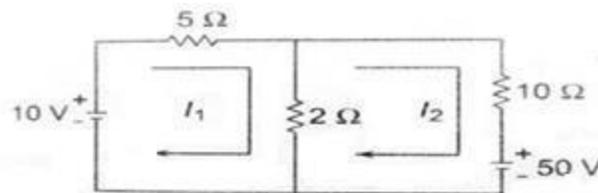
Solution:

By applying KVL to the two meshes, we get

5 I1 + 2(I1- I2)=10

10 I2 + 2(I2- I1)=-50.

Solving the above equations gives ....I1 = 0.25 A and I2 = -4.25 A. The negative sign for the current I2 indicates that it flows in the opposite direction to that assumed in the loop two.

Super Mesh Analysis: If there is only current source between two meshes in the given network then it is difficult to apply the mesh analysis. Because the current source has to be converted into a voltage source in terms of the current source, write down the mesh equations and relate the mesh currents to the current source. But this is a difficult approach .This difficulty can be avoided by creating super mesh which encloses the two meshes that have common current source

Super Mesh: A super mesh is constituted by two adjacent meshes that have a common current source.

Let us illustrate this method with the following simple generalized circuit.
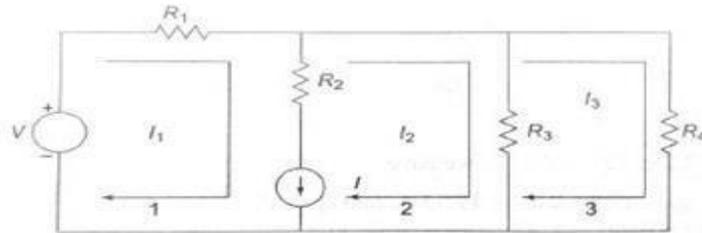
Solution:

Step (1): Identify the position of current source.

Here the current source is common to the two meshes 1 and 2. so, super mesh is nothing but the combination of meshes 1 and 2 .
Step (2): Apply KVL to super mesh and to other meshes
Applying KVL to this super mesh (combination of meshes 1 and 2) we get

R1.I1 + R3 (I2 – I3) = V.............(1)

Applying KVL to mesh 3, we get

R3 (I3 – I2) + R4.I3 = 0 .... (2)

Step (3): Make the relation between mesh currents with current source to get third equation.

 Third equation is nothing but he relation between I , I1 and I2 which is

I1 - I2 = I........... (3)

Step(4): Solve the above equations to get the mesh currents.

Example(1): Determine the current in the 5 Ω resistor shown in the figure below.
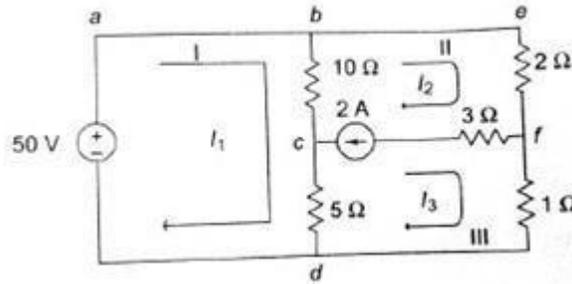
Solution:

Step(1): Here the current source exists between mesh(2) and mesh(3).Hence, super mesh is the combination of mesh(2) and mesh(3) .Applying KVL to the super mesh (combination of mesh 2 and mesh 3 after removing the branch with the current source of 2 A and resistance of 3 Ω) we get :

$10(I2 - I1) + 2.I2 + I3 + 5(I3 - I1) = 0$

$-15.I1 + 12\ I2 + 6.I3 = 0..................$ (1)

Step (2): Applying KVL first to the normal mesh 1 we get :

$10(I1 - I2) + 5(I1 - I3) = 50$

$15.I1 - 10.\ I2 - 5.I3 = 50...................(2)$

**Step (3):** We can get the third equation from the relation between the current source of 2 A , and currents I2 & I3 as :

$I2 - I3 = 2\ A...................$ (3)

Step (4): Solving the above three equations for I1, I2 and I3 we get I1 = 19.99 A I2 = 17.33 A and I3 = 15.33 A

The current in the 5 Ω resistance = I1 - I3 = 19.99 - 15.33 = 4.66 A

Example(2): Write down the mesh equations for the circuit shown in the figure below and find out the values of the currents I1, I2 and I3
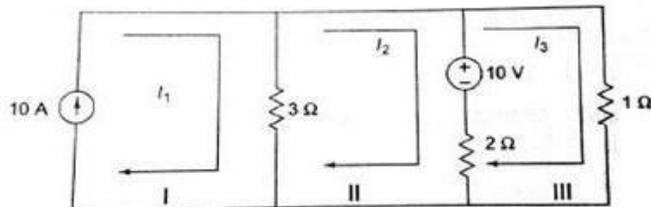
Solution: In this circuit the current source is in the perimeter of the circuit and hence the first mesh is ignored. So, here no need to create the super mesh.

Applying KVL to mesh 1 we get :

$3(I2 – I1) + 2(I2 – I3) = -10$

$-3.I1 +5.I2 – 2.I3 = -10$ ....(1)

Next applying KVL to mesh 2 we get :

$I3 + 2(I3– I2) = 10$

$-2.I2 +3.I3 = -10$ ....(2)

And from the first mesh we observe that....... $I1 = 10$ A..............(3)

And solving these three equations we get : $I1 = 10$ A, $I2 = 7.27$ A, $I3 = 8.18$ A

Nodal analysis:

Nodal analysis provides another general procedure for analyzing circuits nodal voltages as the circuit variables. It is preferably useful for the circuits that have many no. of nodes. It is applicable for the both planar and non planar circuits. This analysis is done by using KCL and Ohm's law.

Node: It is a junction at which two or more branches are interconnected.

Simple Node: Node at which only two branches are interconnected.

Principal Node: Node at which more than two branches are interconnected.

Nodal analysis with example:

Determination of node voltages:
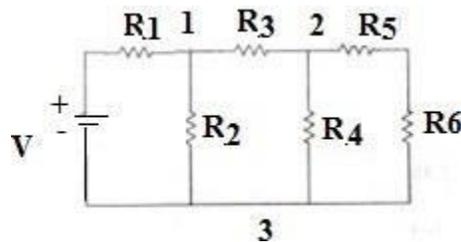
Procedure:

Step (1): Identify the no. nodes, simple nodes and principal nodes in the given circuit. Among all the nodes one node is taken as reference node. Generally bottom is taken as reference node. The potential at the reference node is 0v.

In the given circuit there are 3 principal nodes in which node (3) is the reference node.

Step (2): Assign node voltages to the all the principal nodes except reference node and assign branch currents to all branches.
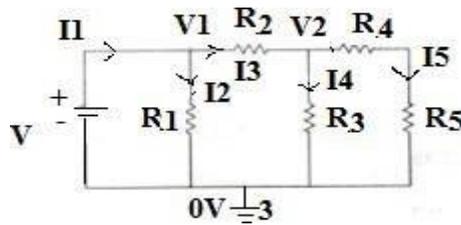
Step (3): Apply KCL to those principal nodes for nodal equations and by using ohm's law express the node voltages in terms of branch current.

Applying KCL to node (1)-----**1=I2+I3**

Using ohm's law, we get   **(V-V1)/R1 =(V2-0)/R2 +(V1-V2)/R3. ....(1)**

Applying KCL to node (2)-----**3=I4 +I5**

Using ohm's law, we get   **(V1-V2)/R3 =(V4-0)/R4 +(V5-0)/R5.............. (2)**

Step(4): Solve the above nodal equations to get the node voltages.

Example: Write the node voltage equations and find out the currents in each branch of the circuit shown in the figure below.
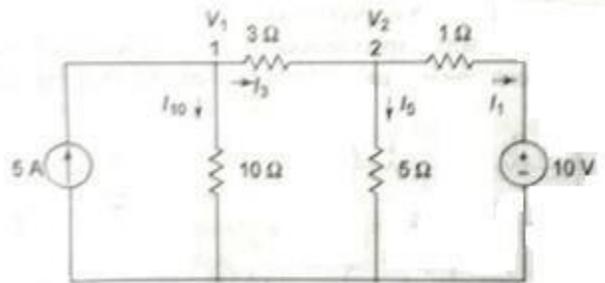
Solution:

The node voltages and the directions of the branch currents are assigned as shown in given figure.
Applying KCL to node 1, we get: **5 = I10+ I3**

$$5= (V1-0)/10 +(V1-V2)/3$$

$$V1(13/30) -V2(1/3) = 5...........(1)$$

Applying KCL to node 2, we get: **I3= I5 + I1**

$$(V1-V2)/3 = (V2 -0)/5 + (V2-10) /1$$

$$V1(1/3)-V2(23/15) = -10...............(2)$$

Solving the these two equations for V1 and V2 we get :

V1 = 19.85 V and V2 = 10.9 V and the currents are :
I10= V1/10 = 1.985A
**I3 = (V1-V2)/3 = (19.85-10.9)/3 = 2.98A**
I5 = V2/5 = 10.9/5 =2.18A
I1 = (V2-10) = (10.9-10)/1 = 0.9A

Super Node Analysis: If there is only voltage source between two nodes in the given network then it is difficult to apply the nodal analysis. Because the voltage source has to be converted into a current source in terms of the voltage source, write down the nodal equations and relate the node voltages to the voltage source. But this is a difficult approach .This difficulty can be avoided by creating super node   which encloses the two nodes that have common voltage source.

Super Node: A super node is constituted by two adjacent nodes that have a common voltage source.

Example: Write the nodal equations by using super node analysis.
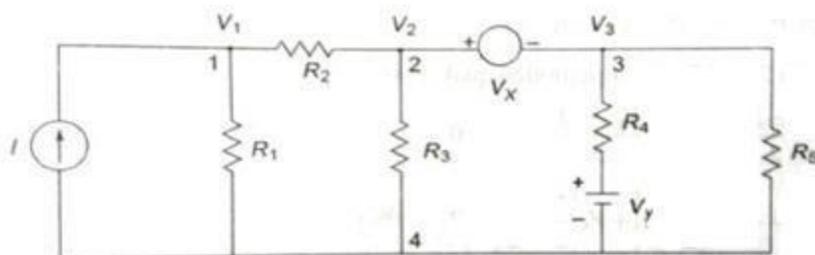
Procedure:

Step(1): Identify the position of voltage source.Here the voltage source is common to the two nodes 2 and 3.so, super node is nothing but the combination of nodes 2 and 3 .

Step (2): Apply KCL to super node and to other nodes.
Applying KCL to this super node (combination of meshes 2 and 3), we get

(V2-V1)/R2 + V2/R3 + (V3-Vy)/R4 + V3/R5 = 0............. (1)

Applying KVL to node 1 ,we get

I = V1/R1 + (V1-V2)/R2 .... (2)

Step (3): Make the relation between node voltages with voltage source to get third equation.

 Third equation is nothing but the relation between VX , V2 and V3 which is
V2 - V3 = Vx .... (3)
Step (4): Solve the above nodal equations to get the node voltages.

Example: Determine the current in the 5 Ω resistor shown in the circuit below
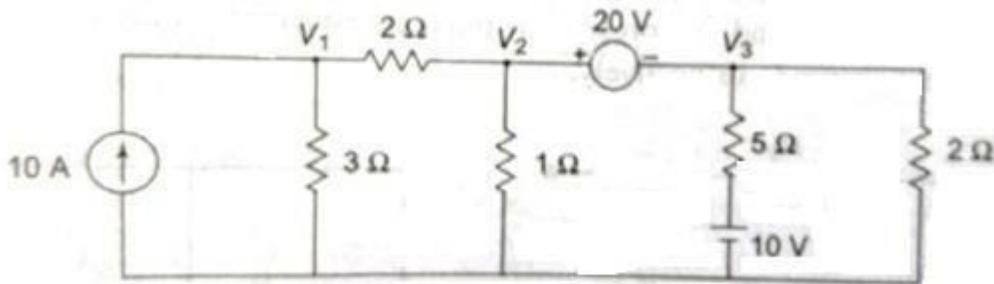
Solution:

Applying KCL to node 1: 10 = V1/3 + (V1-V2)/2
                          V1 [1/3 + 1/2] - V2 /2 = 10

                            0.83 V1 - 0.5 V2 = 10................ (1)

Next applying KCL to the super node2&3 :
(V2-V1)/2 + V2/1 + (V3-10)/5 + V3/2 = 0
-V1/2 + V2(1/2 + 1) V3(1/5 + 1/2) = 2
0.5 V1 + 1.5V2 + 0.7 V3 = 2...................... (2)
and the third and final equation is:
V2 - V3 = 20................... (3)

Solving the above three equations we get V3 = -8.42 V

The current through the 5 Ω resistor I5 = [-8.42 -10] /5 = -3.68 A
The negative sign indicates that the current flows towards the node 3.

UNIT-II
MAGNETIC CIRCUITS

➢ **INTRODUCTION**
➢ **FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION**
➢ **CONCEPT OF SELF & MUTUAL INDUCTANCE**
➢ **DOT CONVENTION**
➢ **COEFFIECIENT OF COUPLING**
➢ **COMPOSTE MAGNETIC CIRCUIT**
➢ **ANALYSIS OF SERIES AND PARALLEL MAGNETIC CIRCUITS**

INTRODUCTION

Although the lines of magnetic flux have no physical existence, they do form a very convenient and useful basis for explaining various magnetic effects and to calculate the magnitudes of various magnetic quantities. The complete closed path followed by any group of magnetic flux lines is referred as magnetic circuit. The lines of magnetic flux never intersect, and each line forms a closed path. Whenever a current is flowing through the coil there will be magnetic flux produced and the path followed by the magnetic flux is known as magnetic circuit. The operation of all the electrical devices like generators, motors, transformers etc. depend upon the magnetism produced by this magnetic circuit. Therefore, to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully.

Magneto Motive Force (MMF)

The magnetic pressure which sets up or tends to set up magnetic flux in a magnetic circuit is known as MMF.

1. Magneto motive force is the measure of the ability of a coil to produce flux.
2. The magnetic flux is due to the existence of the MMF caused by a current flowing through a coil having no. of turns.
3. ∴A coil with 'N' turns carrying a current of 'I' amperes represents a magnetic circuit producing an MMF of NI   MMF=NI
4. Units of MMF = Ampere turns(AT)

Magnetic Flux:

1. The amount of magnetic lines of force set-up in a magnetic circuit is called magnetic flux.
2. The magnetic flux, that is established in a magnetic circuit is proportional to the MMF and the proportional constant is the reluctance of the magnetic circuit.

Magnetic flux α MMF

$$\text{Magnetic flux} = \frac{MMF}{RELUCTANCE} = \frac{NI}{S}$$

3. The unit of magnetic flux is   Weber.

Reluctance:

1. The opposition offered to the flow of magnetic flux in a magnetic circuit is called reluctance

2. Reluctance of a magnetic circuit is defined as the ratio of magneto motive force to the flux established.
3. Reluctance depends upon length(l), area of cross-section(a) and permeability of the material that makes up the magnetic circuit.(S l, S   a, S   l/a)

$$S = \frac{l}{\mu a}$$

$$\text{RELUCTANCE} = \frac{MMF}{FLUX}$$

4. The unit of reluctance is AT/ Wb

Magnetic field strength(H)

1. If the magnetic circuit is homogeneous, and of uniform cross-sectional area, the magnetic field strength is defined as the magneto motive force per unit length of magnetic circuit.

$$H = \frac{MMF}{LENGTH} = \frac{NI}{l}$$

2. The unit of magnetic field strength is AT/m

Magnetic flux density(B)

1. The magnetic flux density in any material is defined as the magnetic flux established per unit area of cross-section.

$$B = \frac{FLUX}{AREA\ OF\ CROSS\ SECTION} = \frac{\emptyset}{A}$$

2. The unit of magnetic flux density is wb/m^2 or TESLA

Relative permeability

1. It is defined as the ratio of flux density established in magnetic material to the flux density established in air or vacuum for the same magnetic field strength.

INTRODUCTION TO ELECTROMAGNETIC INDUCTION::
When a conductor moves in a magnetic field, an *EMF* is generated; when it carries current in a magnetic field, a force is produced. Both of these effects may be deduced from one of the most fundamental principles of electromagnetism, and they provide the basis for a number of devices in which conductors move freely in a magnetic field. It has already been mentioned that most electrical machines employ a different form of construction.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION:
In 1831, Michael Faraday, an English physicist gave one of the most basic laws of electromagnetism called **Faraday's law of electromagnetic induction**. This law explains the working principle of most of the electrical motors, generators, electrical transformers and inductors. This law shows the relationship between electric circuit and magnetic field.

FARADAY'S FIRST LAW
Any change in the magnetic field of a coil of wire will cause an emf to be induced in the coil. This emf induced is called induced emf and if the conductor circuit is closed, the current will also circulate through the circuit and this current is called induced current.
Method to change magnetic field:

➢ By moving a magnet towards or away from the coil
➢ By moving the coil into or out of the magnetic field.
➢ By changing the area of a coil placed in the magnetic field
➢ By rotating the coil relative to the magnet.

Faraday's Second Law
It states that the magnitude of emf induced in the coil is equal to the rate of change of flux that linkages with the coil. The flux linkage of the coil is the product of number of turns in the coil and flux associated with the coil.

Faraday Law Formula
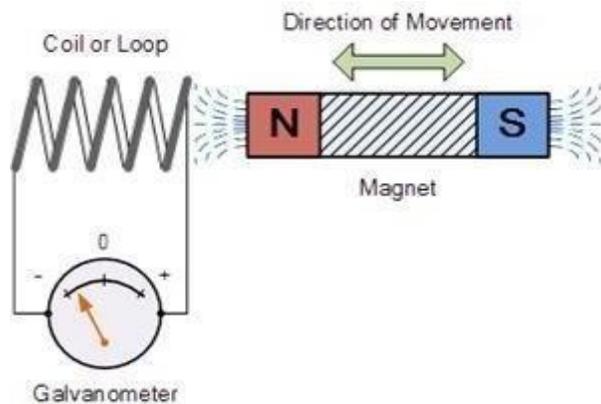
Consider, a magnet is approaching towards a coil. Here we consider two instants at time T_1 and time T_2.
Flux linkage with the coil at time,
$$T_1 = N\phi_1 \ wb$$
Flux linkage with the coil at time,
$$T_2 = N\phi_2 \ wb$$
Change in flux linkage,
$$N(\phi_2 - \phi_1)$$
Let this change in flux linkage be,
$$\phi = (\phi_2 - \phi_1)$$
So, the Change in flux linkage
$$N\phi$$
Now the rate of change of flux linkage
$$\frac{N\phi}{t}$$
Take derivative above equation we will get the rate of change of flux linkage
$$N\frac{d\phi}{dt}$$
But according to Faraday's law of electromagnetic induction, the rate of change of flux linkage is equal to induced emf.
$$E = N\frac{d\phi}{dt}$$
$$E = -N\frac{d\phi}{dt}$$

Where, flux Φ in Wb = B*A

B = magnetic field strength

A = area of the coil

Lenz's law obeys Newton's third law of motion (i.e. to every action there is always an equal and opposite reaction) and the conservation of energy (i.e. energy may neither be created nor destroyed and therefore the sum of all the energies in the system is a constant).

Lenz's law : It states that when an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such, that it produces a current that's magnetic field opposes the change which produces it.

$$E = -N\frac{d\phi}{dt}$$

The negative sign used in Faraday's law of electromagnetic induction, indicates that the induced emf and the change in magnetic flux have opposite signs.

➢ There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either *Fleming's Right-hand Rule*
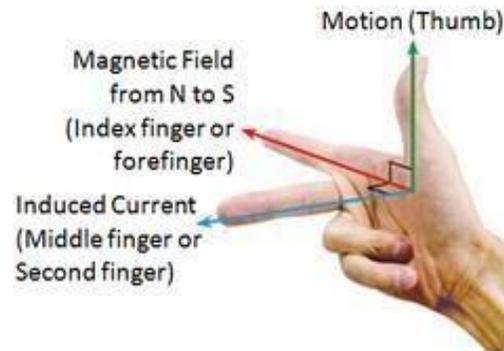
➢ There exists a definite relation between the direction of the induced current, the direction of the flux and the direction of motion of the conductor. The direction of the induced current may be found easily by applying either *Fleming's Left-hand Rule*
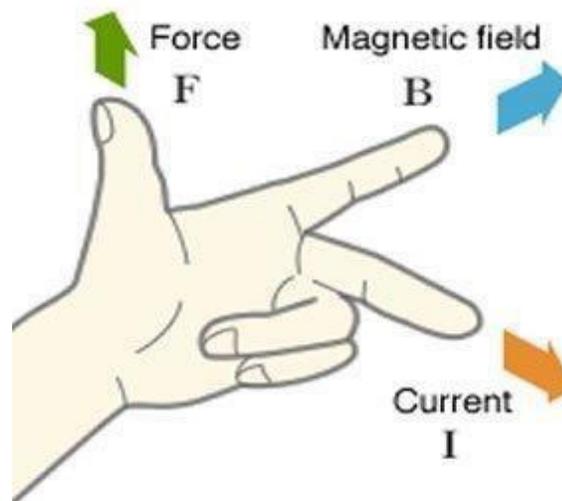
HOW TO INCREASE EMF INDUCED IN A COIL:

➢ By increasing the number of turns in the coil i.e. N, from the formulae derived above it is easily seen that if number of turns in a coil is increased, the induced emf also gets increased.

➢ By increasing magnetic field strength i.e. B surrounding the coil- Mathematically, if magnetic field increases, flux increases and if flux increases emf induced will also get

increased. Theoretically, if the coil is passed through a stronger magnetic field, there will be more lines of force for coil to cut and hence there will be more emf induced.

➢ By increasing the speed of the relative motion between the coil and the magnet - If the relative speed between the coil and magnet is increased from its previous value, the coil will cut the lines of flux at a faster rate, so more induced emf would be produced.

APPLICATIONS OF FARADAY'S LAW:

Faraday law is one of the most basic and important laws of electromagnetism. This law finds its application in most of the electrical machines, industries and medical field etc.

➢ Electrical Transformers work on Faraday's law of mutual induction.

➢ The basic working principle of electrical generator is Faraday's law of electromagnetic induction.

➢ The Induction cooker is a fastest way of cooking. It also works on principle of mutual induction. When current flows through the coil of copper wire placed below a cooking container, it produces a changing magnetic field. This alternating or changing magnetic field induces an emf and hence the current in the conductive container, and we know that flow of current always produces heat in it.

➢ Electromagnetic Flow Meter is used to measure velocity of certain fluids. When a magnetic field is applied to electrically insulated pipe in which conducting fluids are flowing, then according to Faraday's law, an electromotive force is induced in it. This induced emf is proportional to velocity of fluid flowing.

➢ Form the bases of Electromagnetic theory; Faraday's idea of lines of force is used in well known Maxwell's equations. According to Faraday's law, change in magnetic field gives rise to change in electric field and the converse of this is used in Maxwell's equations.

➢ It is also used in musical instruments like electric guitar, electric violin etc.

SELF INDUCTANCE:

Inductance is the property of electrical circuits containing coils in which a change in the electrical current induces an electromotive force (emf). This value of induced emf opposes the change in current in electrical circuits and electric current 'I' produces a magnetic field which generates magnetic flux acting on the circuit containing coils. **The ratio of the magnetic flux to the current is called the self-inductance.**

$$L = \frac{\psi = N\emptyset}{I}$$

The phenomenon of inducing an emf in a coil whenever a current linked with coil changes is called induction. Here units of L are Weber per ampere which is equivalent to Henry.

'Ø' denotes the magnetic flux through the area spanned by one loop, 'I' is the current flowing through the coil and N is the number of loops (turns) in the coil.

MUTUAL INDUCTANCE:

Mutual Inductance is the ratio between induced Electro Motive Force across a coil to the rate of change of current of another adjacent coil in such a way that two coils are in possibility of flux linkage. Mutual induction is a phenomenon when a coil gets induced in EMF across it due to rate of change current in adjacent coil in such a way that the flux of one coil current gets linkage of another coil. Mutual inductance is denoted as (M), it is called co-efficient of Mutual Induction between two coils
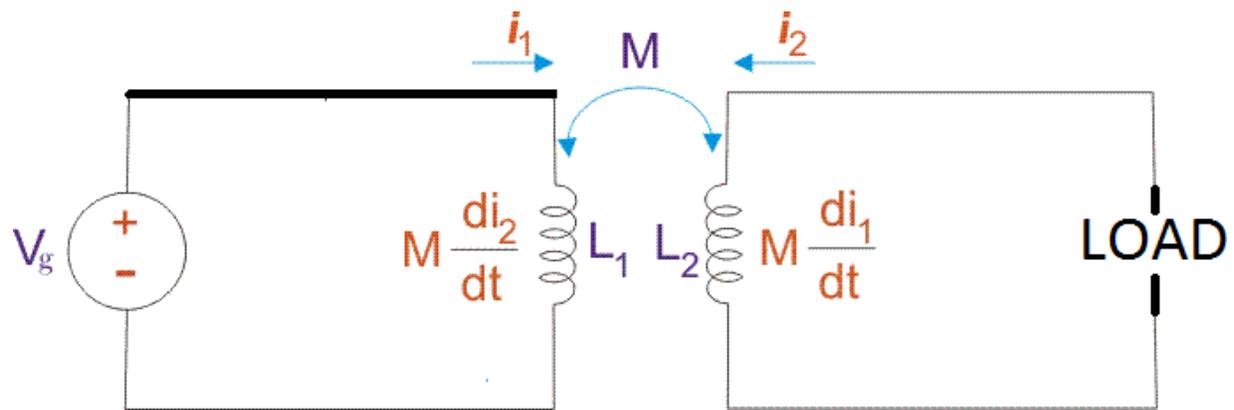
Mutual inductance for two coils gives the same value when they are in mutual induction with each other. Induction in one coil due to its own rate of change of current is called self inductance (L), but due to rate of change of current of adjacent coil it gives **mutual inductance** (M)

From the above figure, first coil carries current i_1 and its self inductance is L_1. Along with its self inductance it has to face mutual induction due to rate of change of current i_2 in the second coil. Same case happens in the second coil also. Dot convention is used to mark the polarity of the mutual induction. Suppose two coils are placed nearby
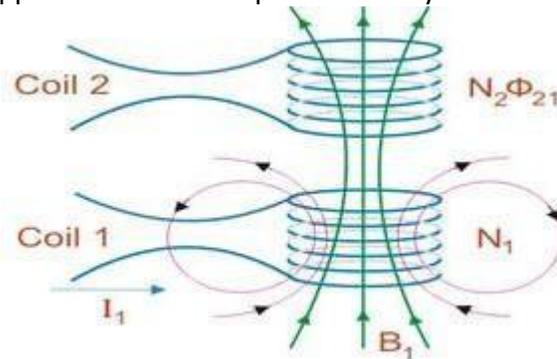
Coil 1 carries I_1 current having N_1 number of turn. Now the flux density created by the coil 1 is B_1. Coil 2 with N_2 number of turn gets linked with this flux from coil 1. So flux linkage in coil 2 is N_2.

ϕ_{21} [ϕ_{21} is called leakage flux in coil 2 due to coil 1].

$$\varepsilon_2 = -N_2 \cdot \frac{d\varphi_{21}}{dt} \; volt.$$

$$Again, \quad \varepsilon_2 = -M_{21} \cdot \frac{dl_1}{dt} \; volt.$$

Now it can be written from these equations,

$$M_{21} = \frac{\varphi_{21} N_2}{I_1}$$

DOT CONVENTION:

➢ Dot convention is used to determine the polarity of a magnetic coil in respect of other magnetic coil.

➢ Dot convention is normally used to determine the total or equivalent inductance (L_{eq}).

SERIES AIDING:
  ➢ Suppose two coils are in series with same place dot.
  ➢ When 2 dots are at the same place of both inductors(while at entering place or leaving place)as shown in below figure i.e. the total mutual inductance gets aided(added)
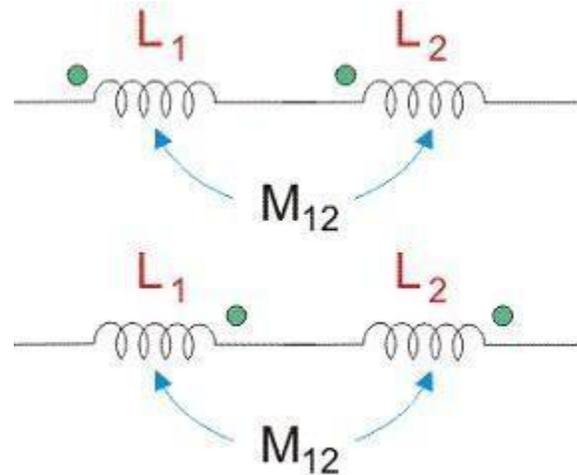

Mutual inductance between them is positive.

$$So, \quad L_{eq} = L_1 + L_2 + 2M_{12}$$

SERIES OPPOSING:

  ➢ Suppose two coils are in series with opposite place dot.
  ➢ When 2 dots are at the opposite place of both inductors(while one at entering place and other at leaving place)as shown in below figure i.e. the total mutual inductance gets differed
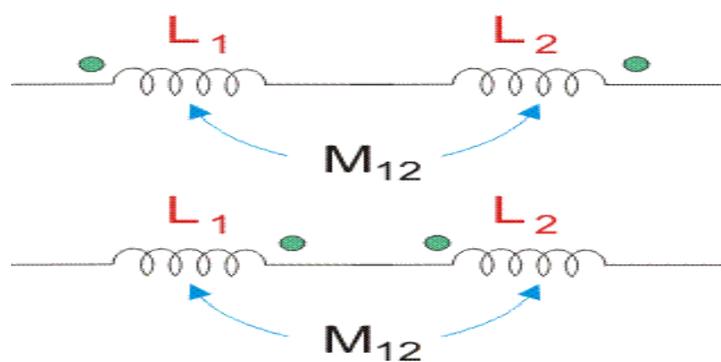
Mutual inductance between them is negative.

$$So, \quad L_{eq} = L_1 + L_2 - 2M_{12}$$

PARALLEL AIDING:
  ➢ Suppose two coils are in parallel with same place dot.
  ➢ When 2 dots are at the same place of both inductors(while at entering place or leaving place)as shown in below figure i.e. the total mutual inductance gets aided(added)

$$V = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$V = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix} \begin{bmatrix} \dfrac{di_1}{dt} \\ \dfrac{di_2}{dt} \end{bmatrix} = \begin{bmatrix} V \\ V \end{bmatrix}$$

$$\Delta = \begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix} = L_1 L_2 - M^2$$

$$\frac{di_1}{dt} = \frac{\begin{vmatrix} V & M \\ V & L_2 \end{vmatrix}}{\Delta} = \frac{V(L_2 - M)}{\Delta} \quad , \quad \frac{di_2}{dt} = \frac{\begin{vmatrix} L_1 & V \\ M & V \end{vmatrix}}{\Delta} = \frac{V(L_1 - M)}{\Delta}$$

From the above figure,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{V(L_2 - M)}{\Delta} + \frac{V(L_1 - M)}{\Delta} = \frac{V(L_1 + L_2 - 2M)}{\Delta} = \frac{V(L_1 + L_2 - 2M)}{L_1 L_2 - M^2}$$

$$V = \frac{L_1 L_2 - M^2}{(L_1 + L_2 - 2M)} \frac{di}{dt}$$

Therefore total inductance is given by,

$$L_{eq} = \frac{L_1 L_2 - M^2}{(L_1 + L_2 - 2M)}$$

PARALLEL OPPOSING:

➤ Suppose two coils are in parallel with opposite place dot.
➤ When 2 dots are at the opposite place of both inductors(while one at entering place and other at leaving place)as shown in below figure i.e. the total mutual inductance gets differed

$$V = L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt}$$

$$V = L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}$$

$$\begin{bmatrix} L_1 & -M \\ -M & L_2 \end{bmatrix} \begin{bmatrix} \dfrac{di_1}{dt} \\ \dfrac{di_2}{dt} \end{bmatrix} = \begin{bmatrix} V \\ V \end{bmatrix}$$

$$\Delta = \begin{vmatrix} L_1 & -M \\ -M & L_2 \end{vmatrix} = L_1 L_2 - M^2$$

$$\frac{di_1}{dt} = \frac{\begin{vmatrix} V & -M \\ V L_2 \end{vmatrix}}{\Delta} = \frac{V(L_2 + M)}{\Delta}, \quad \frac{di_2}{dt} = \frac{\begin{vmatrix} L_1 & V \\ -M & V \end{vmatrix}}{\Delta} = \frac{V(L_1 + M)}{\Delta}$$

From the above figure,

$$i = i_1 + i_2$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} = \frac{V(L_2 + M)}{\Delta} + \frac{V(L_1 + M)}{\Delta} = \frac{V(L_1 + L_2 + 2M)}{\Delta} = \frac{V(L_1 + L_2 + 2M)}{L_1 L_2 - M^2}$$

$$= \frac{L_1 L_2 - M^2}{V(L_1 + L_2 + 2M)} \frac{di}{dt}$$

Therefore total inductance is given by,

$$L_{eq} = \frac{L_1 L_2 - M^2}{(L_1 + L_2 + 2M)}$$

COEFFICIENT OF COUPLING:

The fraction of magnetic flux produced by the current in one coil that links with the other coil is called **coefficient of coupling** between the two coils. It is denoted by (k).
Two coils are taken coil A and coil B, when current flows through one coil it produces flux; the whole flux may not link with the other coil coupled, and this is because of leakage flux by a fraction (k) known as **Coefficient of Coupling.**
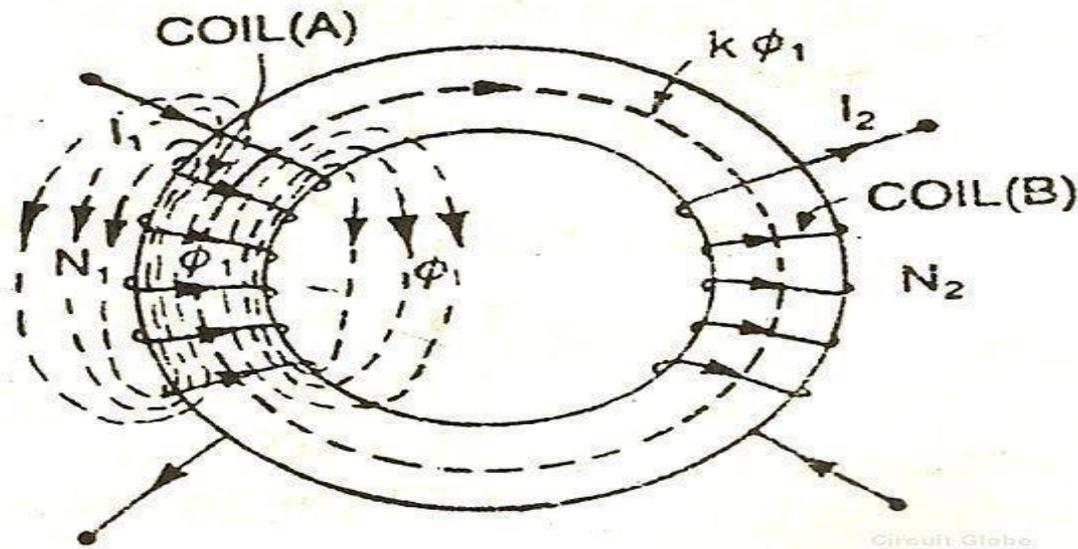
k=1 when the flux produced by one coil completely links with the other coil and is called magnetically tightly coupled.

k=0 when the flux produced by one coil does not link at all with the other coil and thus the coils are said to be magnetically isolated.

DERIVATION:

Consider two magnetic coils A and B.
When current I_1 flows through coil A.

$$L_1 = \frac{N_1 \varphi_1}{I_1} \ and \ M = \frac{N_2 \varphi_{12}}{I_1} \ldots\ldots (1) \ as \ (\varphi_{12} = k\varphi_1)$$

Considering coil B in which current I_2 flows

$$L_2 = \frac{N_2 \varphi_2}{I_2} \ and \ M = \frac{N_1 \varphi_{21}}{I_2} = \frac{N_1 k\varphi_2}{I_2} \ldots\ldots (2) \ as \ (\varphi_2 = k\varphi_2)$$

Multiplying equation (1) and (2)

$$MxM = \frac{N_{2k\varphi_1}}{I_1} \, x \, \frac{N_{1k\varphi_2}}{I_2}$$

$$M^2 = k^2 \frac{N_{1\varphi_1}}{I_1} \, x \, \frac{N_{2\varphi_2}}{I_2} = k^2 L_1 L_2$$

$$M = \sqrt{L_1 L_2} \quad \ldots\ldots\ldots (A)$$

The above equation (A) shows the relationship between mutual inductance and self inductance between two the coils

SERIES MAGNETIC CIRCUIT:

➢ A series magnetic circuit is analogous to a series electric circuit. A magnetic circuit is said to be series, if the same flux is flowing through all the elements connected in a magnetic circuit. Consider a circular ring having a magnetic path of 'l' meters, area of cross section 'a' m^2 with a mean radius of 'R' meters having a coil of 'N' turns carrying a current of 'I' amperes wound uniformly as shown in below fig
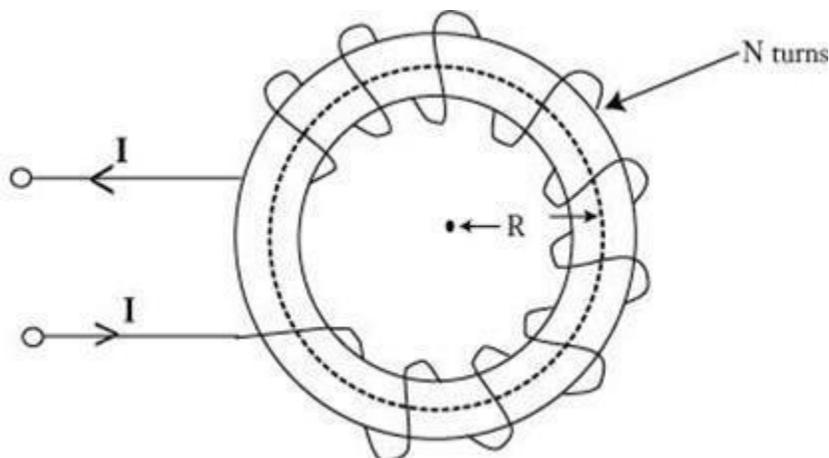
The flux produced by the circuit is given by

$$\text{Magnetic flux} = \frac{MMF}{RELUCTANCE} = \frac{N}{S} = \frac{NI}{l/\mu a}$$

In the above equation NI is the MMF of the magnetic circuit, which is analogous to EMF in the electrical circuit.

PARALLEL MAGNETIC CIRCUIT

➢ A magnetic circuit which has more than one path for magnetic flux is called a parallel magnetic circuit. It can be compared with a parallel electric circuit which has more than one path for electric current. The concept of parallel magnetic circuit is illustrated in fig. **2.** Here a coil of 'N' turns wounded on limb 'AF' carries a current of 'I' amperes. The magnetic flux 'ϕ_1' set up by the coil divides at 'B' into two paths namely

Magnetic flux passes 'ϕ_2' along the path 'BE'
Magnetic flux passes 'ϕ_3' along the path 'BCDE' i.e $\phi_1 = \phi_2 + \phi_3$
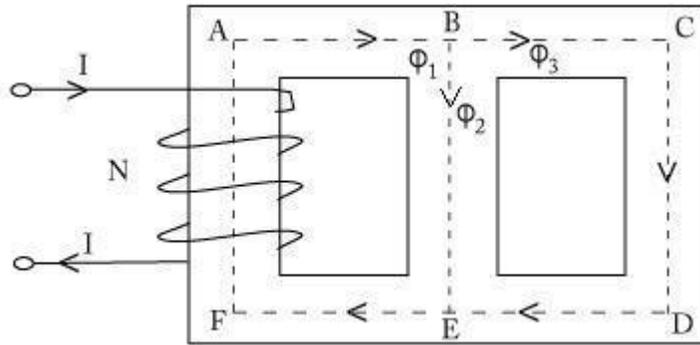
The magnetic paths 'BE' and 'BCDE' are in parallel and form a parallel magnetic circuit. The AT required for this parallel circuit is equal to AT required for any one of the paths. Let S_1= reluctance of path EFAB

> Let, S_1= reluctance of path EFAB
>
> S_2= reluctance of path BE
>
> S_3= reluctance of path BCDE
>
> Total MMF= MMF for path EFAB + MMF for path BE or path BCD
>
> $NI = \Phi_1 S_1 + \Phi_2 S_2 = \Phi_1 S_1 + \Phi_3 S_3$

COMPOSITE MAGNETIC CIRCUIT:

Consider a magnetic circuit which consists of two specimens of iron arranged as shown in figure. Let ℓ_1 and ℓ_2 be the mean lengths of specimen 1 and specimen2 in meters, A_1 and A_2 be their respective cross sectional areas in square meters, and $_1$ and $_2$ be their respective relative permeability's.
The reluctance of specimen 1 is given as

$$S_1 = \frac{\ell_1}{\mu_0 \mu_1 A_1} \quad (AT/Wb)$$

and that for specimen 2 is

$$S_2 = \frac{\ell_2}{\mu_0 \mu_2 A_2} \quad (AT/Wb)$$
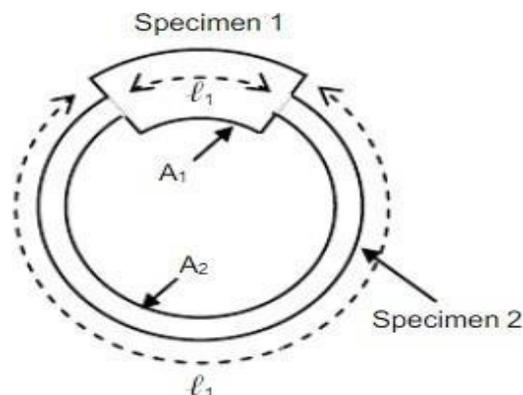
If a coil of N turns carrying a current I is wound on the specimen 1 and if the magnetic flux is assumed to be confined to iron core then the total reluctance is given by the sum of the individual reluctances S_1 and S_2. This is equivalent to adding the resistances of a series circuit. Thus the total reluctance is given by

$$S = S_1 + S_2 = \frac{\ell_1}{\mu_0 \mu_1 A_1} + \frac{\ell_2}{\mu_0 \mu_2 A_2} \quad AT/Wb$$

And the total flux is given by

$$\Phi = \frac{mmf}{S} = \frac{NI}{\dfrac{\ell_1}{\mu_0 \mu_1 A_1} + \dfrac{\ell_2}{\mu_0 \mu_2 A_2}} \quad \left(\frac{AT}{(AT/Wb)} \Rightarrow Wb \right)$$

UNIT-3 : SINGLE PHASE AC CIRCUITS

- Average value, RMS Value, Form factor and Peak factor for different Periodic waveforms.
- J-Notation, Complex and Polar forms of Representation.
- Steady state Analysis of Series RLC circuits.
- Concept of Reactance, Impedance, Susceptance and Admittance.
- Phase and phase difference
- Concept of power factor, real, reactive and complex power

Average value, RMS Value, Form factor and Peak factor for different waveforms:

Sinusoidal wave:

A sinewave is defined by the trigonometric sine function. When plotted as voltage (V) as a function of phase (θ), it looks similar to the figure to the below. The waveform repeats every 2p radians (360°), and is symmetrical about the voltage axis (when no DC offset is present). Voltage and current exhibiting cyclic behavior is referred to as alternating; i.e., alternating current (AC). One full cycle is shown here. The basic equation for a sinewave is as follows:

$$V(\theta) = V_{pk} \cdot \sin(\theta)$$

There are a number of ways in which the amplitude of a sinewave is referenced, usually as peak voltage (V_{pk} or V_p), peak-to-peak voltage (V_{pp} or V_{p-p} or V_{pkpk} or V_{pk-pk}), average voltage (V_{av} or V_{avg}), and root-mean-square voltage (V_{rms}). Peak voltage and peak-to-peak voltage are apparent by looking at the above plot. Root-mean-square and average voltage are not so apparent.

Average Voltage (V_{avg})

As the name implies, V_{avg} is calculated by taking the average of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the sinewave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through p/2 radians (90°).

As with the V_{rms} formula, a full derivation for the V_{avg} formula is given here as well.

$$V_{avg} = \frac{1}{\pi/2} \cdot \int_0^{\pi/2} V_{pk} \cdot \sin\theta \cdot d\theta = \frac{2}{\pi} \cdot V_{pk} \cdot -\cos\theta \Big|_0^{\pi/2}$$

$$= \frac{-2}{\pi} \cdot V_{pk} \cdot \left(\cos\frac{\pi}{2} - \cos 0 \right) = \frac{-2}{\pi} \cdot V_{pk} \cdot (0-1) = \frac{2}{\pi} \cdot V_{pk}$$

So, $V_{avg} = \dfrac{2}{\pi} \cdot V_{pk} \approx 0.636\ V_{pk,}$

Root-Mean-Square Voltage (V_{rms})

As the name implies, V_{rms} is calculated by taking the square root of the mean average of the square of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the sine wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through p/2 radians (90°).

V_{rms} is the value indicated by the vast majority of AC voltmeters. It is the value that, when applied across a resistance, produces that same amount of heat that a direct current (DC) voltage of the same magnitude would produce. For example, 1 V applied across a 1 Ω resistor produces 1 W of heat. A 1 V_{rms} sine wave applied across a 1 Ω resistor also produces 1 W of heat. That 1 V_{rms} sine wave has a peak voltage of √2 V (≈1.414 V), and a peak-to-peak voltage of 2√2 V (≈2.828 V).

Since finding a full derivation of the formulas for root-mean-square (V_{rms}) voltage is difficult, it is done here for you.

$$V_{rms\,(sinewave)} = \sqrt{\dfrac{1}{\pi/2} \int_{0}^{\pi/2} (V_{pk}\sin\theta)^2 \cdot d\theta} = \sqrt{\dfrac{2 \cdot V_{pk}^2}{\pi}\left(\dfrac{\theta}{2} - \dfrac{1}{4}\sin(2\cdot\theta)\right)\Big|_{0}^{\pi/2}}$$

$$= \dfrac{\sqrt{2} \cdot V_{pk}}{\sqrt{\pi}} \sqrt{\left(\dfrac{\pi/2}{2} - \dfrac{1}{4}\sin(\pi)\right) - \left(\dfrac{0}{2} - \dfrac{1}{4}\sin(0)\right)}$$

$$= \dfrac{\sqrt{2} \cdot V_{pk}}{\sqrt{\pi}} \sqrt{\left(\dfrac{\pi}{4} - 0\right) - (0 - 0)} = \dfrac{\sqrt{2} \cdot V_{pk}}{\sqrt{\pi}} \cdot \dfrac{\sqrt{\pi}}{2} = \dfrac{1}{\sqrt{2}} \cdot V_{pk}$$

So, $V_{rms} = \dfrac{1}{\sqrt{2}} \cdot V_{pk} \approx 0.707\ V_{pk,}$

Form factor:

Two alternating periodic waveforms of the same amplitude and frequency may look different depending upon their wave shape/form and then their average & RMS values will be different. In order to compare such different waveforms of the same frequency and amplitude but of different wave shape a parameter called Form factor is defined as the ratio of it's RMS and Average values.

For a sinusoidal signal of peak voltage V_m it is given by :

Form factor of a sinusoidal signal = V_{rms} /V_{av}
= 0.707 V_m / 0.637 V_m = 1.11

Peak Factor (Or Crest factor): Is defined as the ratio of maximum value to the R.M.S value of an alternating quantity.

$$\text{Peak factor of a sinusoidal signal} = V_{max}/V_{rms}$$
$$= V_{max}/(0.707\ V_m)$$
$$= 1.414$$

Triangular wave:
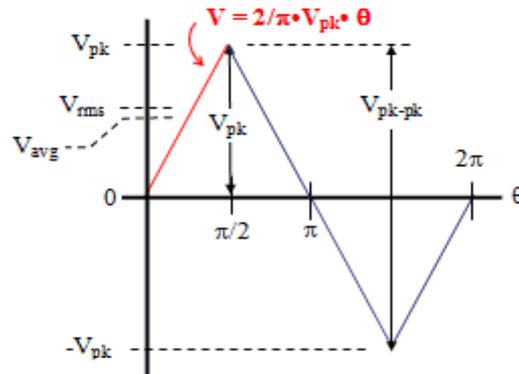
When plotted as voltage (V) as a function of phase (θ), a triangle wave looks similar to the figure to the above. The waveform repeats every 2π radians (360°), and is symmetrical about the voltage axis (when no DC offset is present). Voltage and current exhibiting cyclic behavior is referred to as alternating; i.e., alternating current (AC). One full cycle is shown here. The basic equation for a triangle wave is as follows:

$$V = \frac{2}{\pi} \cdot V_{pk} \cdot \theta \quad \text{for } 0 \le \theta < \pi/2$$

There are a number of ways in which the amplitude of a triangle wave is referenced, usually as peak voltage (V_{pk} or V_p), peak-to-peak voltage (V_{pp} or V_{p-p} or V_{pkpk} or V_{pk-pk}), average voltage (V_{av} or V_{avg}), and root-mean-square voltage (V_{rms}). Peak voltage and peak-to-peak voltage are apparent by looking at the above plot. Root-mean-square and average voltage are not so apparent.

Average Voltage (V_{avg})

As the name implies, V_{avg} is calculated by taking the average of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the triangle wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through $\pi/2$ radians (90°).

As with the V_{rms} formula, a full derivation for the V_{avg} formula is given here as well.

$$V_{avg} = \frac{1}{\pi/2} \int_0^{\pi/2} \frac{2}{\pi} \cdot V_{pk} \cdot \theta \cdot d\theta = \frac{2}{\pi} \cdot \frac{2}{\pi} \cdot V_{pk} \cdot \frac{1}{2} \cdot \theta^2 \Big|_0^{\pi/2}$$

$$= \frac{4}{\pi^2} \cdot V_{pk} \cdot \frac{1}{2} \cdot \left(\frac{\pi^2}{4} - 0\right) = \frac{1}{2} \cdot V_{pk}$$

$$V_{avg} = \frac{1}{2} \cdot V_{pk}$$
$$\approx 0.5\ V_{pk}$$

Root-Mean-Square Voltage (V$_{rms}$)

      As the name implies, V$_{rms}$ is calculated by taking the square root of the mean average of the square of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the triangle wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through π/2 radians (90°).

      V$_{rms}$ is the value indicated by the vast majority of AC voltmeters. It is the value that, when applied across a resistance, produces that same amount of heat that a direct current (DC) voltage of the same magnitude would produce. For example, 1 V applied across a 1 Ω resistor produces 1 W of heat. A 1 V$_{rms}$ triangle wave applied across a 1 Ω resistor also produces 1 W of heat. That 1 V$_{rms}$ triangle wave has a peak voltage of √3 V (≈1.732 V), and a peak-to-peak voltage of 2√3 V (≈3.464 V).

      Since finding a full derivation of the formulas for root-mean-square (V$_{rms}$) voltage is difficult, it is done here for you.

$$V_{rms} = \sqrt{\frac{1}{\pi/2} \cdot \int_0^{\pi/2} \left(\frac{2}{\pi} \cdot V_{pk} \cdot \theta\right)^2 d\theta} = \sqrt{\frac{2}{\pi} \cdot \frac{4}{\pi^2} \cdot V_{pk}^2 \cdot \int_0^{\pi/2} \theta^2 d\theta} = \sqrt{\frac{8}{\pi^3} \cdot V_{pk}^2 \cdot \int_0^{\pi/2} \theta^2 d\theta}$$

$$= \sqrt{\frac{8}{\pi^3} \cdot V_{pk}^2 \cdot \frac{1}{3} \cdot \theta^3 \Big|_0^{\pi/2}} = \sqrt{\frac{8}{3 \cdot \pi^3} \cdot V_{pk}^2 \cdot \left[\left(\frac{\pi}{2}\right)^3 - 0\right]}$$

$$= \sqrt{\frac{8}{3 \cdot \pi^3} \cdot V_{pk}^2 \cdot \frac{\pi^3}{2^3}} = \sqrt{\frac{1}{3} \cdot V_{pk}^2} = \frac{1}{\sqrt{3}} \cdot V_{pk}$$

So, $V_{rms} = \frac{1}{\sqrt{3}} \cdot V_{pk} \approx 0.577\, V_{pk}$

Form factor:
      Form factor of a triangular signal = V$_{rms}$ /V$_{av}$
               =.577Vpk/.5Vpk
               =1.15

Peak Factor (Or Crest factor): Is defined as the ratio of maximum value to the R.M.S value

      Peak factor of a triangular signal=V$_{pk}$/V$_{rms}$
               =Vpk/.577Vpk
               =1.732
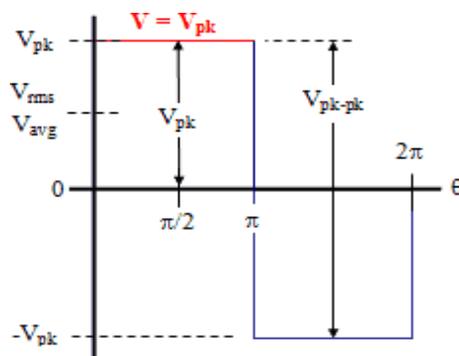
Square wave:

When plotted as voltage (V) as a function of phase (θ), a square wave looks similar to the figure to the above. The waveform repeats every 2π radians (360°), and is symmetrical about the voltage axis (when no DC offset is present). Voltage and current exhibiting cyclic behavior is referred to as alternating; i.e., alternating current (AC). One full cycle is shown here.

The basic equation for a square wave is as follows:

$$V_{one\ complete\ cycle} = \begin{vmatrix} 1, & \text{for } 0 \leq \theta < \pi \\ -1, & \text{for } \pi \leq \theta < 2\pi \end{vmatrix}$$

There are a number of ways in which the amplitude of a square wave is referenced, usually as peak voltage (V_{pk} or V_p), peak-to-peak voltage (V_{pp} or V_{p-p} or V_{pkpk} or V_{pk-pk}), average voltage (V_{av} or V_{avg}), and root-mean-square voltage (V_{rms}). Peak voltage and peak-to-peak voltage are apparent by looking at the above plot. Root-mean-square and average voltage are not so apparent.

Average Voltage (V_{avg})

As the name implies, V_{avg} is calculated by taking the average of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the square wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through π/2 radians (90°).

As with the V_{rms} formula, a full derivation for the V_{avg} formula is given here as well.

$$Vavg = \frac{1}{\pi/2} \cdot \int_0^{\pi/2} V_{pk} \cdot d\theta = \frac{2}{\pi} \cdot V_{pk} \cdot \theta \Big|_0^{\pi/2}$$
$$= \frac{2}{\pi} \cdot V_{pk} \cdot \left(\frac{\pi}{2} - 0 \right) = V_{pk}$$

So, $V_{avg} = V_{pk}$

Root-Mean-Square Voltage (V_{rms})

As the name implies, V_{rms} is calculated by taking the square root of the mean average of the square of the voltage in an appropriately chosen interval. In the case of symmetrical waveforms like the square wave, a quarter cycle faithfully represents all four quarter cycles of the waveform. Therefore, it is acceptable to choose the first quarter cycle, which goes from 0 radians (0°) through π/2 radians (90°).

V_{rms} is the value indicated by the vast majority of AC voltmeters. It is the value that, when applied across a resistance, produces that same amount of heat that a direct current (DC) voltage of the same magnitude would produce. For example, 1 V applied across a 1 Ω resistor produces 1 W of heat. A 1 V_{rms} square wave applied across a 1 Ω resistor also produces 1 W of heat. That 1 V_{rms} square wave has a peak voltage of 1 V, and a peak-to-peak voltage of 2 V.

Since finding a full derivation of the formulas for root-mean-square (V_{rms}) voltage is difficult, it is done here for you.

$$V_{rms} = \sqrt{\frac{1}{\pi/2} \cdot \int_0^{\pi/2} V_{pk}^2 \cdot d\theta} = \sqrt{\frac{2}{\pi} \cdot V_{pk}^2 \cdot \theta \Big|_0^{\pi/2}}$$

$$= \sqrt{\frac{2}{\pi} \cdot V_{pk}^2 \cdot \left(\frac{\pi}{2} - 0\right)} = \sqrt{V_{pk}^2} = V_{pk}$$

So, $V_{rms} = V_{pk}$

Form factor:

Form factor of a triangular signal = V_{rms} / V_{av}

=Vpk/Vpk

=1

Peak Factor (Or Crest factor):

Peak factor of a triangular signal=V_{pk}/V_{rms}

=Vpk/Vpk

J notation:

The mathematics used in Electrical Engineering to add together resistances, currents or DC voltages use what are called "real numbers" either as integers or as fractions.But real numbers are not the only kind of numbers we need to use especially when dealing with frequency dependent sinusoidal sources and vectors. As well as using normal or real numbers, Complex Numbers were introduced to allow complex equations to be solved with numbers that are the square roots of negative numbers, $\sqrt{-1}$.

In electrical engineering this type of number is called an "imaginary number" and to distinguish an imaginary number from a real number the letter " j " known commonly in electrical engineering as the j-operator, is used. The letter j is placed in front of a real number to signify its imaginary number operation.

Examples of imaginary numbers are: j3, j12, j100 etc. Then a complex number consists of two distinct but very much related parts, a " Real Number " plus an " Imaginary Number ".Complex Numbers represent points in a two dimensional complex or s-plane that are referenced to two distinct axes. The horizontal axis is called the "real axis" while the vertical axis is called the "imaginary axis". The real and imaginary parts of a complex number are abbreviated as Re(z) and Im(z), respectively.

Complex numbers that are made up of real (the active component) and imaginary (the reactive component) numbers can be added, subtracted and used in exactly the same way as elementary algebra is used to analyse dc circuitsThe rules and laws used in mathematics for the addition or subtraction of imaginary numbers are the same as for real numbers, j2 + j4 = j6 etc. The only difference is in multiplication because two imaginary numbers multiplied together becomes a negative real number. Real numbers can also be thought of as a complex number but with a zero imaginary part labelled j0.

The j-operator has a value exactly equal to $\sqrt{-1}$, so successive multiplication of " j ", (j x j) will result in j having the following values of, -1, -j and +1. As the j-operator is commonly used to indicate the anticlockwise rotation of a vector, each successive multiplication or power of " j ", j^2, j^3 etc, will force the vector to rotate through an angle of 90° anticlockwise

as shown below. Likewise, if the multiplication of the vector results in a -j operator then the phase shift will be -90º, i.e. a clockwise rotation.

Vector Rotation

90^0 rotation: $\quad j^1 = \sqrt{-1} = +j$

180^0 rotation: $\quad j^2 = \left(\sqrt{-1}\right)^2 = -1$

270^0 rotation: $\quad j^3 = \left(\sqrt{-1}\right)^3 = -j$

360^0 rotation: $\quad j^4 = \left(\sqrt{-1}\right)^4 = +1$
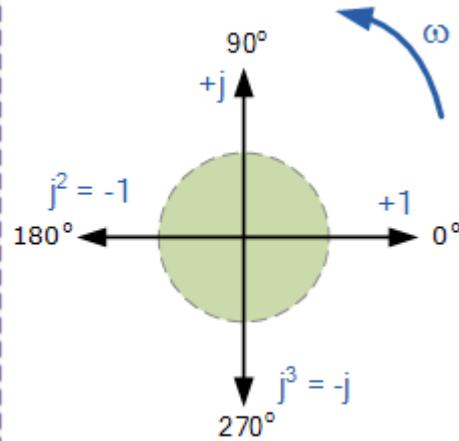
So by multiplying an imaginary number by j^2 will rotate the vector by 180º anticlockwise, multiplying by j^3 rotates it 270º and by j^4 rotates it 360º or back to its original position. Multiplication by j^{10} or by j^{30} will cause the vector to rotate anticlockwise by the appropriate amount. In each successive rotation, the magnitude of the vector always remains the same.

Complex and Polar forms of Representation:

In Electrical Engineering there are different ways to represent a complex number either graphically or mathematically. One such way that uses the cosine and sine rule is called the Cartesian or Rectangular Form.

A complex number is represented by a real part and an imaginary part that takes the generalised form of:

$$Z=x+jy$$

Where

Z - is the Complex Number representing the Vector

x - is the Real part or the Active component

y - is the Imaginary part or the Reactive component

j - is defined by $\sqrt{-1}$

In the rectangular form, a complex number can be represented as a point on a two-dimensional plane called the complex or s-plane. So for example, Z = 6 + j4 represents a single point whose coordinates represent 6 on the horizontal real axis and 4 on the vertical imaginary axis as shown.

Complex Numbers using the Complex or s-plane:
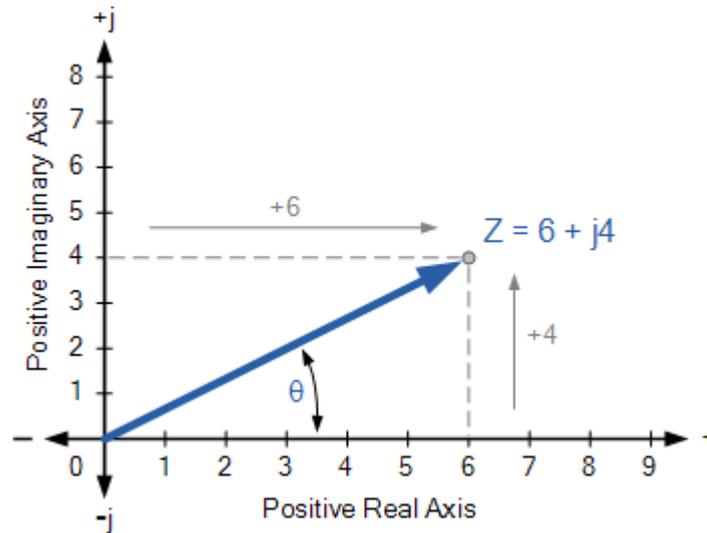
Complex Numbers using Polar Form:

Unlike rectangular form which plots points in the complex plane, the Polar Form of a complex number is written in terms of its magnitude and angle. Thus, a polar form vector is presented as: Z = A ∠±θ, where: Z is the complex number in polar form, A is the magnitude or modulo of the vector and θ is its angle or argument of A which can be either positive or negative. The magnitude and angle of the point still remains the same as for the rectangular form above, this time in polar form the location of the point is represented in a "triangular form" as shown below.

Polar Form Representation of a Complex Number:
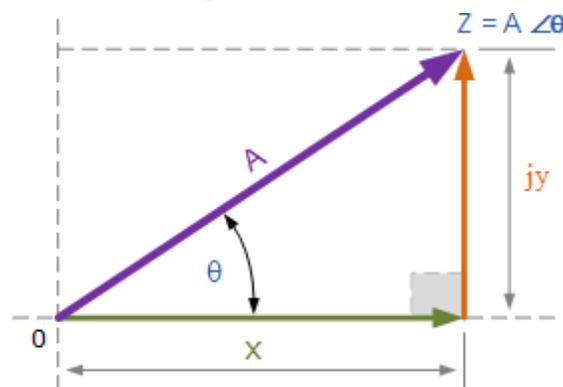
As the polar representation of a point is based around the triangular form, we can use simple geometry of the triangle and especially trigonometry and Pythagoras's Theorem on triangles to find both the magnitude and the angle of the complex number. As we remember from school, trigonometry deals with the relationship between the sides and the angles of triangles so we can describe the relationships between the sides as:

$$A^2=X^2+Y^2$$

$$A=\sqrt{X^2+Y^2}$$

Also $X=A\cos\theta$ $Y=A\sin\theta$

Using trigonometry again, the angle θ of A is given as follows.
$$\Theta=\tan^{-1}y/x$$

Then in Polar form the length of A and its angle represents the complex number instead of a point. Also in polar form, the conjugate of the complex number has the same magnitude or modulus it is the sign of the angle that changes, so for example the conjugate of $6\angle30^\circ$ would be $6\angle-30^\circ$.

Steady state Analysis of Series RLC circuits:
Thus far we have seen that the three basic passive components: resistance (R), inductance (L), and capacitance (C) have very different phase relationships to each other when connected to a sinusoidal AC supply.
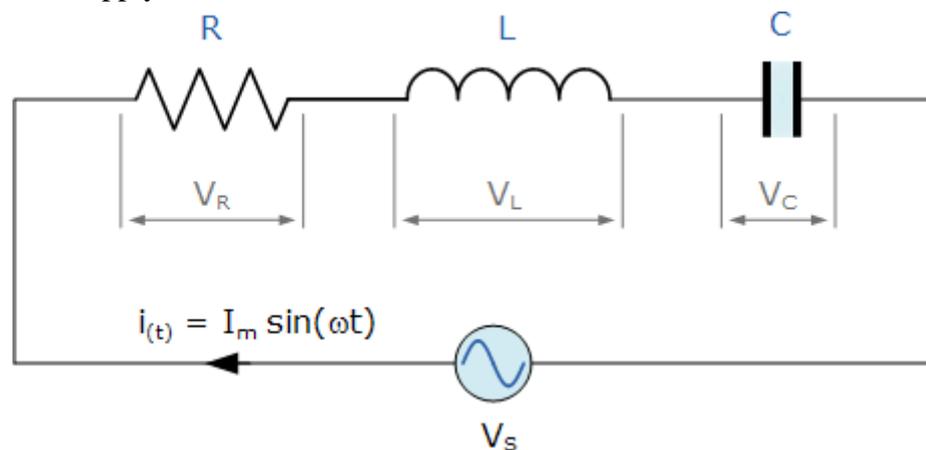
In a pure ohmic resistor the voltage waveforms are "in-phase" with the current. In a pure inductance the voltage waveform "leads" the current by 90°, giving us the expression of: ELI. In a pure capacitance the voltage waveform "lags" the current by 90°, giving us the expression of: ICE.

This phase difference, Φ depends upon the reactive value of the components being used and hopefully by now we know that reactance, (X) is zero if the circuit element is resistive, positive if the circuit element is inductive and negative if it is capacitive thus giving their resulting impedances as:

Element Impedance:

Circuit element	Resistsnce(R)	Reactance(X)	Impeadance(Z)
RESISTOR	R	0	Z_R=R RL0^0
INDUCTOR	L	WL	Z_L=WL L90^0
CAPACITOR	C	1/WC	Z_C=1/WC L-90^0

The series RLC circuit above has a single loop with the instantaneous current flowing through the loop being the same for each circuit element. Since the inductive and capacitive reactance's X_L and X_C are a function of the supply frequency, the sinusoidal response of a series RLC circuit will therefore vary with frequency, f. Then the individual voltage drops across each circuit element of R, L and C element will be "out-of-phase" with each other as defined by:

$$i_{(t)} = I_{max} \sin(\omega t)$$

The instantaneous voltage across a pure resistor, V_R is "in-phase" with current
The instantaneous voltage across a pure inductor, V_L "leads" the current by 90º
The instantaneous voltage across a pure capacitor, V_C "lags" the current by 90º
Therefore, V_L and V_C are 180º "out-of-phase" and in opposition to each other
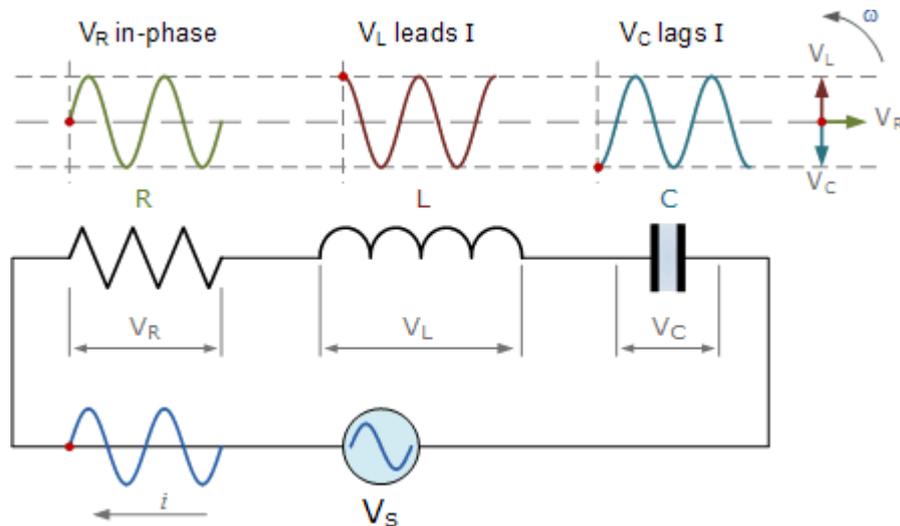
The amplitude of the source voltage across all three components in a series RLC circuit is made up of the three individual component voltages, V_R, V_L and V_C with the current common to all three components. The vector diagrams will therefore have the current vector as their reference with the three voltage vectors being plotted with respect to this reference as shown below.

Individual Voltage Vectors
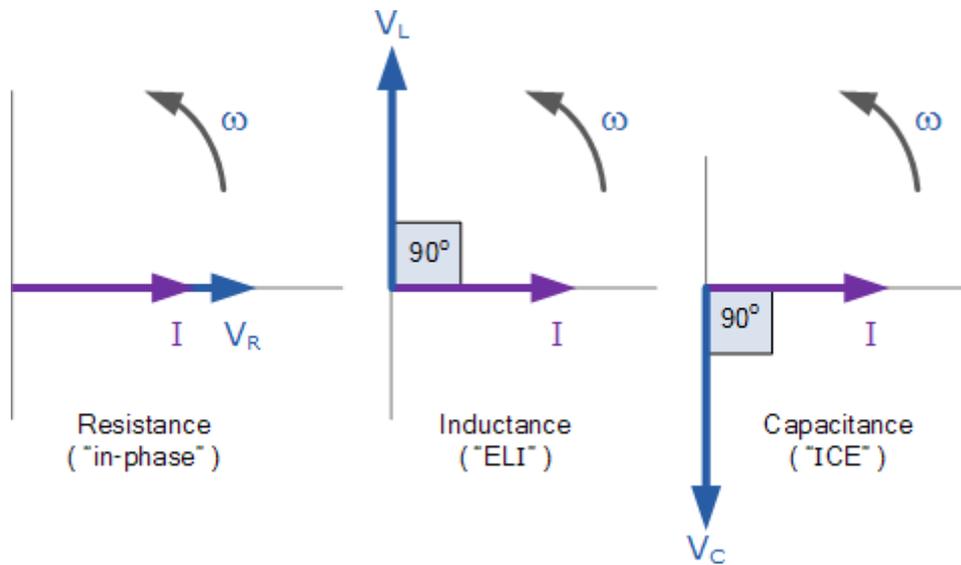
This means then that we cannot simply add together V_R, V_L and V_C to find the supply voltage, V_S across all three components as all three voltage vectors point in different directions with regards to the current vector. Therefore we will have to find the supply voltage, V_S as the Phasor Sum of the three component voltages combined together vectorially.

Kirchoff's voltage law (KVL) for both loop and nodal circuits states that around any closed loop the sum of voltage drops around the loop equals the sum of the EMF's. Then applying this law to the these three voltages will give us the amplitude of the source voltage, V_S as.

Instantaneous Voltages for a Series RLC Circuit:

$$\text{KVL:} \quad V_S - V_R - V_L - V_C = 0$$

$$V_S - IR - L\frac{di}{dt} - \frac{Q}{C} = 0$$

$$\therefore V_S = IR + L\frac{di}{dt} + \frac{Q}{C}$$

The phasor diagram for a series RLC circuit is produced by combining together the three individual phasors above and adding these voltages vectorially. Since the current flowing through the circuit is common to all three circuit elements we can use this as the reference vector with the three voltage vectors drawn relative to this at their corresponding angles.

The resulting vector V_S is obtained by adding together two of the vectors, V_L and V_C and then adding this sum to the remaining vector V_R. The resulting angle obtained between V_S and i will be the circuits phase angle as shown below.

Phasor Diagram for a Series RLC Circuit:
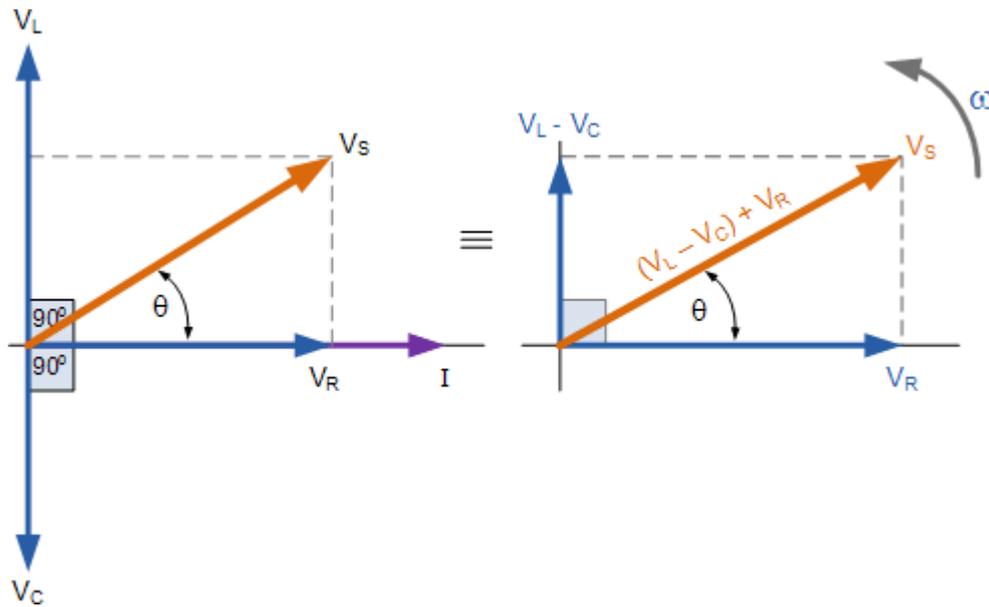
We can see from the phasor diagram on the right hand side above that the voltage vectors produce a rectangular triangle, comprising of hypotenuse V_S, horizontal axis V_R and vertical axis $V_L - V_C$ Hopefully you will notice then, that this forms our old favourite the Voltage Triangle and we can therefore use Pythagoras's theorem on this voltage triangle to mathematically obtain the value of V_S as shown.

Voltage Triangle for a Series RLC Circuit:

$$V_S^2 = V_R^2 + (V_L - V_C)^2$$

$$V_S = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Please note that when using the above equation, the final reactive voltage must always be positive in value, that is the smallest voltage must always be taken away from the largest voltage we cannot have a negative voltage added to V_R so it is correct to have $V_L - V_C$ or $V_C - V_L$. The smallest value from the largest otherwise the calculation of V_S will be incorrect.We know from above that the current has the same amplitude and phase in all the components of a series RLC circuit. Then the voltage across each component can also be described mathematically according to the current flowing through, and the voltage across each element as.

$$V_R = iR \sin(\omega t + 0^\circ) = i.R$$

$$V_L = iX_L \sin(\omega t + 90^\circ) = i.j\omega L$$

$$V_C = iX_C \sin(\omega t - 90^\circ) = i.\frac{1}{j\omega C}$$

By substituting these values into Pythagoras's equation above for the voltage triangle will give us:

$$V_R = I.R \quad V_L = I.X_L \quad V_C = I.X_C$$

$$V_S = \sqrt{(I.R)^2 + (I.X_L - I.X_C)^2}$$

$$V_S = I.\sqrt{R^2 + (X_L - X_C)^2}$$

$$\therefore V_S = I \times Z \quad \text{where: } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

So we can see that the amplitude of the source voltage is proportional to the amplitude of the current flowing through the circuit. This proportionality constant is called the Impedance of the circuit which ultimately depends upon the resistance and the inductive and capacitive reactance's.

Then in the series RLC circuit above, it can be seen that the opposition to current flow is made up of three components, X_L, X_C and R with the reactance, X_T of any series RLC circuit being defined as: $X_T = X_L - X_C$ or $X_T = X_C - X_L$ with the total impedance of the circuit being thought of as the voltage source required to drive a current through it.

The Impedance of a Series RLC Circuit

As the three vector voltages are out-of-phase with each other, X_L, X_C and R must also be "out-of-phase" with each other with the relationship between R, X_L and X_C being the vector sum of these three components thereby giving us the circuits overall impedance, Z. These circuit impedance's can be drawn and represented by an Impedance Triangle as shown below.

The Impedance Triangle for a Series RLC Circuit
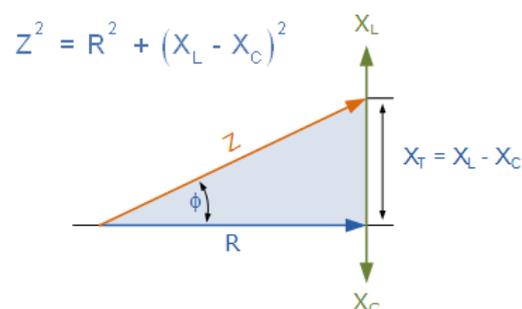
The impedance Z of a series RLC circuit depends upon the angular frequency, ω as do X_L and X_C If the capacitive reactance is greater than the inductive reactance, $X_C > X_L$ then the overall circuit reactance is capacitive giving a leading phase angle.

Likewise, if the inductive reactance is greater than the capacitive reactance, $X_L > X_C$ then the overall circuit reactance is inductive giving the series circuit a lagging phase angle. If the two reactance's are the same and $X_L = X_C$ then the angular frequency at which this occurs is called the resonant frequency and produces the effect of resonance

Then the magnitude of the current depends upon the frequency applied to the series RLC circuit. When impedance, Z is at its maximum, the current is a minimum and likewise, when Z is at its minimum, the current is at maximum. So the above equation for impedance can be re-written as:

$$\text{Impedance, } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The phase angle, θ between the source voltage, V_S and the current, i is the same as for the angle between Z and R in the impedance triangle. This phase angle may be positive or negative in value depending on whether the source voltage leads or lags the circuit current and can be calculated mathematically from the ohmic values of the impedance triangle as:

$$\cos\phi = \frac{R}{Z} \qquad \sin\phi = \frac{X_L - X_C}{Z} \qquad \tan\phi = \frac{X_L - X_C}{R}$$

Series RLC Circuit Example

A series RLC circuit containing a resistance of 12Ω, an inductance of 0.15H and a capacitor of 100uF are connected in series across a 100V, 50Hz supply. Calculate the total circuit impedance, the circuits current, power factor and draw the voltage phasor diagram
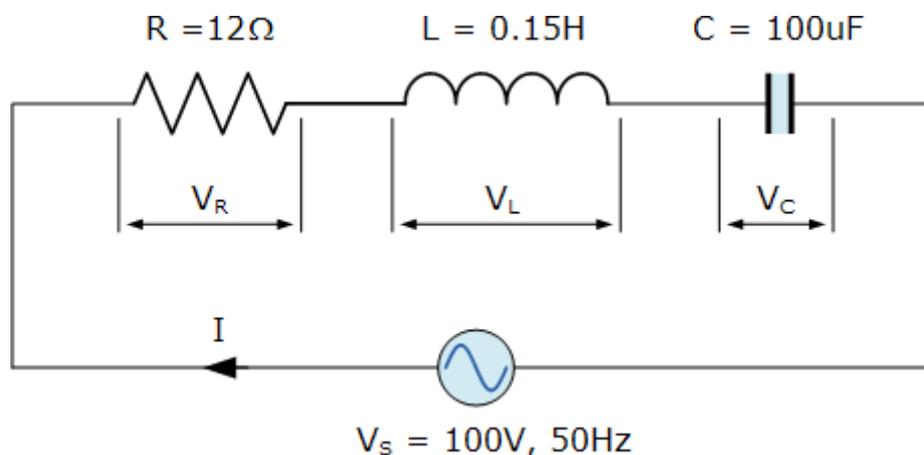
Inductive Reactance, X_L.

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.15 = 47.13\Omega$$

Capacitive Reactance, X_C.

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.83\Omega$$

Circuit Impedance, Z

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{12^2 + (47.13 - 31.83)^2}$$

$$Z = \sqrt{144 + 234} = 19.4\Omega$$

Circuits Current, I.

$$I = \frac{V_S}{Z} = \frac{100}{19.4} = 5.14\,Amps$$

Voltages across the Series RLC Circuit, V_R, V_L, V_C.

$$V_R = I \times R = 5.14 \times 12 = 61.7\,volts$$

$$V_L = I \times X_L = 5.14 \times 47.13 = 242.2\,volts$$

$$V_C = I \times X_C = 5.14 \times 31.8 = 163.5\,volts$$

Circuits Power factor and Phase Angle, θ.

$$\cos\phi = \frac{R}{Z} = \frac{12}{19.4} = 0.619$$

$$\therefore \cos^{-1} 0.619 = 51.8^0 \text{ lagging}$$

Phasor Diagram.
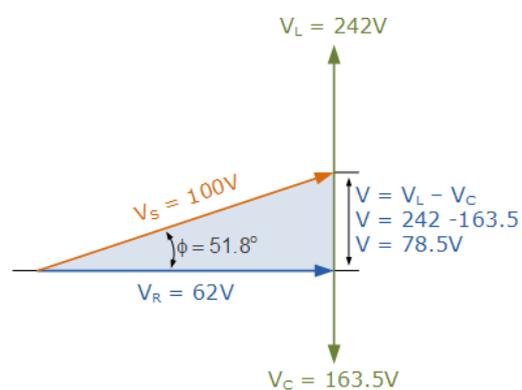
Concept of Reactance, Impedance, Susceptance and Admittance:

Reactance is essentially inertia against the motion of electrons. It is present anywhere electric or magnetic fields are developed in proportion to applied voltage or current, respectively; but most notably in capacitors and inductors. When alternating current goes through a pure reactance, a voltage drop is produced that is 90° out of phase with the current. Reactance is mathematically symbolized by the letter "X" and is measured in the unit of ohms (Ω).

Impedance is a comprehensive expression of any and all forms of opposition to electron flow, including both resistance and reactance. It is present in all circuits, and in all components. When alternating current goes through an impedance, a voltage drop is produced that is somewhere between 0° and 90° out of phase with the current. Impedance is mathematically symbolized by the letter "Z" and is measured in the unit of ohms (Ω), in complex form

Admittance is also a complex number as impedance which is having a real part, Conductance (G) and imaginary part, Susceptance (B).

$$Y = G + jB$$
$$Y \rightarrow Admittance\ in\ Siemens$$
$$G \rightarrow Conductance\ in\ Siemens = \frac{R}{R^2 + X^2}$$
$$B \rightarrow Susceptance\ in\ Siemens = -\frac{X}{R^2 + X^2}$$

(it is negative for capacitive susceptance and positive for inductive susceptance)

$$j^2 = -1$$
$$|Y| = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}}$$
$$\angle Y = arctan\left(\frac{B}{G}\right) = arctan\left(-\frac{X}{R}\right)$$

Susceptance (symbolized B) is an expression of the ease with which alternating current (AC) passes through a capacitance or inductance

Phase and phase difference:
Generally all sinusoidal waveforms will not pass exactly through the zero axis point at the same time, but may be "shifted" to the right or to the left of 0° by some value when compared to another sine wave. Any sine wave that does not pass through zero at t = 0 has a phase shift.

The phase difference or phase shift as it is also called of a Sinusoidal Waveform is the angle Φ (Greek letter Phi), in degrees or radians that the waveform has shifted from a certain reference point along the horizontal zero axis. In other words phase shift is the lateral

difference between two or more waveforms along a common axis and sinusoidal waveforms of the same frequency can have a phase difference.

The phase difference, Φ of an alternating waveform can vary from between 0 to its maximum time period, T of the waveform during one complete cycle and this can be anywhere along the horizontal axis between, Φ = 0 to 2π (radians) or Φ = 0 to 360° depending upon the angular units used.

Phase difference can also be expressed as a *time shift* of τ in seconds representing a fraction of the time period, T for example, +10mS or – 50uS but generally it is more common to express phase difference as an angular measurement.

Then the equation for the instantaneous value of a sinusoidal voltage or current waveform we developed in the previous Sinusoidal Waveform will need to be modified to take account of the phase angle of the waveform and this new general expression becomes.

Phase Difference Equation

$$A_{(t)} = A_{max} \times \sin(\omega t \pm \Phi)$$

Where:

A_m - is the amplitude of the waveform.

ωt - is the angular frequency of the waveform in radian/sec.

Φ (phi) - is the phase angle in degrees or radians that the waveform has shifted either left or right from the reference point

Phase Relationship of a Sinusoidal Waveform:

In-phase (φ = 0°) $A_{(t)} = A_m \sin(\omega t)$

Positive Phase (+φ) $A_{(t)} = A_m \sin(\omega t + \phi)$

Negative Phase (-φ) $A_{(t)} = A_m \sin(\omega t - \phi)$
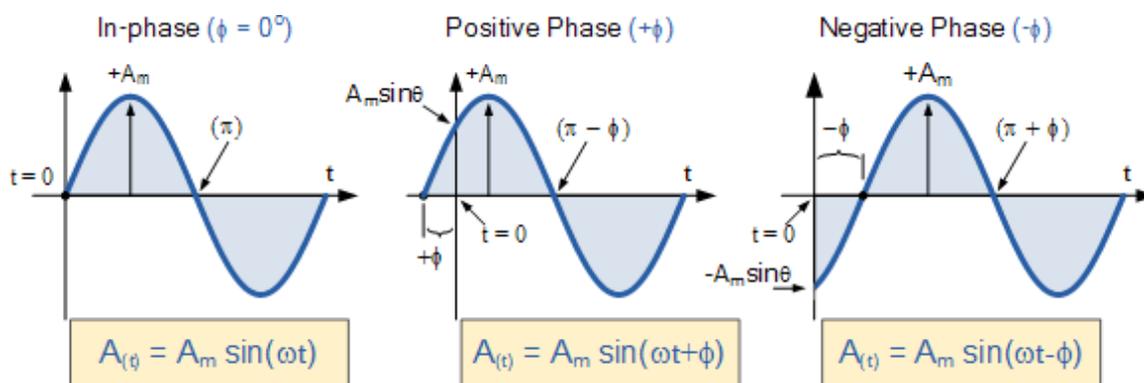
Two Sinusoidal Waveforms – "in-phase"
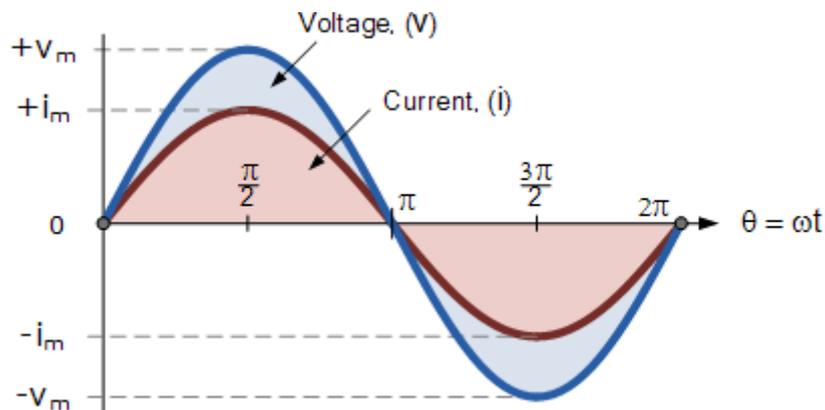
Phase Difference of a Sinusoidal Waveform:
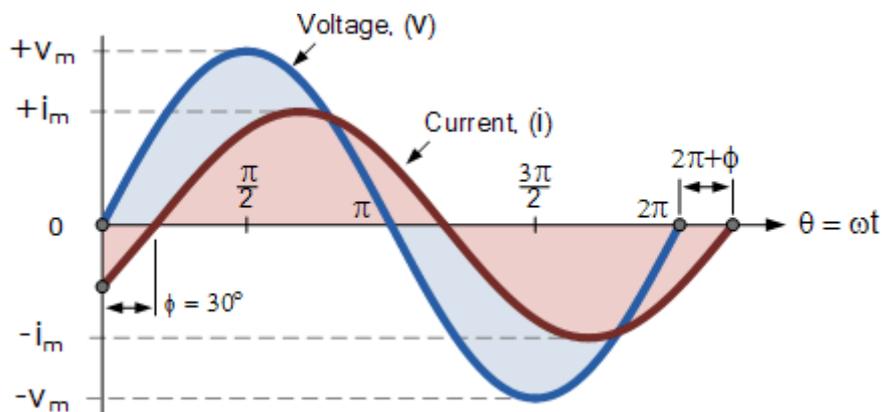
The voltage waveform above starts at zero along the horizontal reference axis, but at that same instant of time the current waveform is still negative in value and does not cross this reference axis until 30° later. Then there exists a Phase difference between the two waveforms as the current cross the horizontal reference axis reaching its maximum peak and zero values after the voltage waveform.

As the two waveforms are no longer "in-phase", they must therefore be "out-of-phase" by an amount determined by phi, Φ and in our example this is 30°. So we can say that the two waveforms are now 30° out-of phase. The current waveform can also be said to be "lagging" behind the voltage waveform by the phase angle, Φ. Then in our example above the two waveforms have a Lagging Phase Difference so the expression for both the voltage and current above will be given as.

$$\text{Voltage,} (v_t) = V_m \sin \omega t$$

$$\text{Current,} (i_t) = I_m \sin(\omega t - \theta)$$

where, i lags v by angle Φ

Likewise, if the current, i has a positive value and crosses the reference axis reaching its maximum peak and zero values at some time before the voltage, v then the current waveform will be "leading" the voltage by some phase angle. Then the two waveforms are said to have a Leading Phase Difference and the expression for both the voltage and the current will be.

$$\text{Voltage}, (v_t) = V_m \sin \omega t$$

$$\text{Current}, (i_t) = I_m \sin(\omega t + \theta)$$

where, i leads v by angle Φ

Concept of power factor, real, reactive and complex power:

Complex Power is defined as the product of Voltage phasor and conjugate of current phasor

If S is the complex power then,

$$S = V \cdot I^*$$

V is the phasor representation of voltage and I* is the conjugate of current phasor.

So if V is the reference phasor then V can be written as $|V| \angle 0$.
(Usually one phasor is taken reference which makes zero degrees with real axis. It eliminates the necessity of introducing a non zero phase angle for voltage)
Let current lags voltage by an angle φ, so $I = |I| \angle\text{-}\varphi$
(current phasor makes -φ degrees with real axis)

$$I^* = |I| \angle\varphi$$

So,

$$S = |V| \; |I| \angle(0+\varphi) = |V| \; |I| \angle\varphi$$

(For multiplication of phasors we have considered polar form to facilitate calculation)

Writing the above formula for S in rectangular form we get

$$S = |V| \; |I| \cos \varphi \; + \; j \; |V| \; |I| \sin \varphi$$

The real part of complex power S is $|V| |I| \cos \varphi$ which is the real power or average power and the imaginary part $|V| |I| \sin \varphi$ is the reactive power.

So, $S = P + j Q$

Where $P = |V| |I| \cos \varphi$ and $Q = |V| |I| \sin \varphi$

P is measured in watt and Q is measured in VoltAmp-Reactive or VAR. In power systems instead of these smaller units larger units like Megawatt, MVAR and MVA is used.

The ratio of real power and apparent power is the power factor

$$\text{power factor} = \text{Cos } \varphi = |P| \, / \, |S|$$

$$= |P| \, / \, \sqrt{(P^2 + Q^2)}$$

• Locus Diagrams with variation of various parameters
• Series RC and RL circuits
• Parallel RLC circuits
• Resonance
• Series and Parallel circuits
• Concept of Bandwidth and Quality factor

Locus Diagrams with variation of various parameters:

Introduction: In AC electrical circuits the magnitude and phase of the current vector depends upon the values of R,L&C when the applied voltage and frequency are kept constant. The path traced by the terminus (tip) of the current vector when the parameters R,L&C are varied is called the current
Locus diagram . Locus diagrams are useful in studying and understanding the behavior of the RLC circuits when one of these parameters is varied keeping voltage and frequency constant.
In this unit,Locus diagrams are developed and explained for series RC,RL circuits and Parallel LC circuits along with their internal resistances when the parameters R,L and C are varied.
The term circle diagram identifies locus plots that are either circular or semicircular. The defining equations of such circle diagrams are also derived in this unit for series RC and RL diagrams.
In both series RC,RL circuits and parallel LC circuits resistances are taken to be in series with L and C to highlight the fact that all practical L and C components will have at least a small value of internal resistance.

Series RL circuit with varying Resistance R:

Refer to the series RL circuit shown in the figure (a) below with constant X_L and varying R. The current I_L lags behind the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to V/X_L and lies along the I axis with phase angle equal to 90^0. When R is increased from zero to infinity the current gradually reduces from V/X_L to 0 and phase angle also reduces from 90^0 to 0^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve I axis.
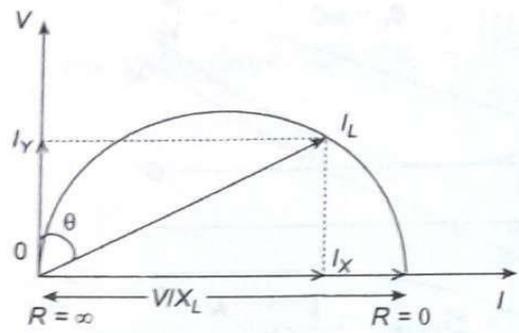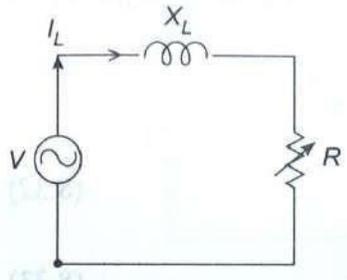
**Fig(a): Series RL circuit with   Fig(b): Locus of current vector I_L with
                    variation of RVarying Resistance R**

The related equations are:

$I_L = V/Z$ $\sin\Theta = X_L/Z$ or $Z = X_L/\sin\Theta$ and $\cos\Theta = R / Z$ Therefore $I_L = (V/X_L)\sin\Theta$

For constant V and X_L the above expression for I_L is the polar equation of a circle with diameter (V/X_L) as shown in the figure above.

Circle equation for the RL circuit: (with fixed reactance and variable Resistance):

The X and Y coordinates of the current I_L are $I_X = I_L\sin\Theta$ $I_Y = I_L\cos\Theta$
From the relations given above and earlier we get

$$I_X = (V/Z)(X_L/Z) = V X_L/Z^2 \text{----------------------(1)}$$

and

$$I_Y = (V/Z)(R/Z) = V R/Z^2 \text{------------------------------(2)}$$

Squaring and adding the above two equations we get

$$I_X^2 + I_Y^2 = V^2(X^2+R^2) / Z^4 = (V^2 Z^2)/ Z^4 = V^2/Z^2 \text{----------------(3)}$$

From equation (1) above we have $Z^2 = V X_L / I_X$ and substituting this in the above equation (3) we get :

$$I_X^2 + I_Y^2 = V^2/ (V_L X_X I) = (V_L/X) I$$
$$\text{or} I_X^2 + I_Y^2 - (V/X_L) I$$
$$= 0$$

Adding $(V/2X_L)^2$ to both sides, the above equation can be written as

$$[I_X - V/2X_L]^2 + I_Y^2 = (V/2X_L)^2 \text{--------------------------------(4)}$$

Equation (4) above represents a circle with a radius of $(V/2X_L)$ and with it's coordinates of the centre as $(V/2X_L, 0)$

Series RC circuit with varying Resistance R:

Refer to the series RC circuit shown in the figure (a) below with constant X_C and varying R. The current I_C leads the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When R=0 we can see that the current is maximum equal to $-V/X_C$ and lies along the negative I axis with phase angle equal to -90^0. When R is increased from zero to infinity the current gradually reduces from $-V/X_C$ to 0 and phase angle also reduces from -90^0 to 0^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle but now with its diameter along the negative I axis.

Circle equation for the RC circuit: (with fixed reactance and variable Resistance):

In the same way as we got for the Series RL circuit with varying resistance we can get the circle equation for an RC circuit with varying resistance as :

$$[I_X + V/2X_C]^2 + I_Y^2 = (V/2X_C)^2$$

whose coordinates of the centre are $(-V/2X_C, 0)$ and radius equal to $V/2X_C$
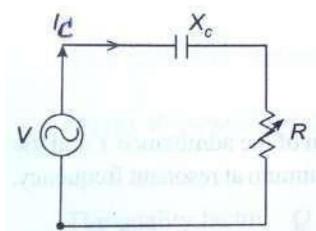

Fig: Series RC circuit with
Varying Resistance R
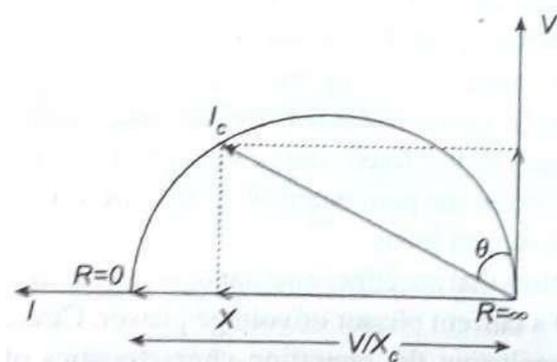
Fig: Locus of current vector I_c
with variation of

RSeries RL circuit with varying Reactance X_L:

Refer to the series RL circuit shown in the figure (a) below with constant R and varying X_L. The current I_L lags behind the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_L/R)$ for a given value of R as shown in the figure (b) below. When $X_L = 0$ we can see that the current is maximum equalto V/R and lies along the +ve V axis with phase angle equal to 0^0. When X_L is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to 90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis and on to its right side.
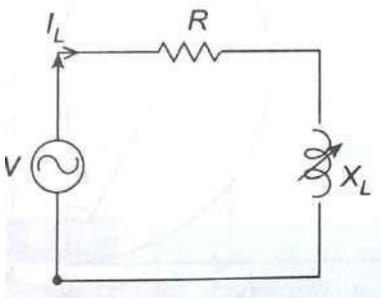
Fig(a): Series RL circuit with varying X_L
variation of X_L
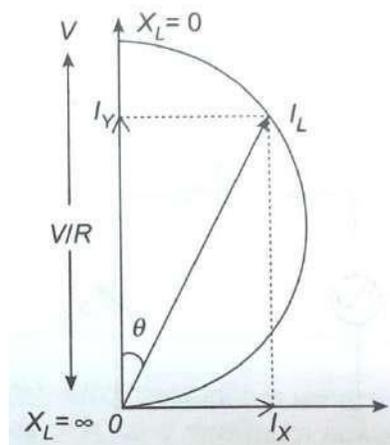
Fig(b) : Locus of current vector I_L with

Series RC circuit with varying Reactance X_C:

Refer to the series RC circuit shown in the figure (a) below with constant R and varying X_C. The current I_C leads the applied voltage V by a phase angle $\Theta = \tan^{-1}(X_C/R)$ for a given value of R as shown in the figure (b) below. When $X_C = 0$ we can see that the current is maximum equal to V/R and lies along the V axis with phase angle equal to 0^0. When X_C is increased from zero to infinity the current gradually reduces from V/R to 0 and phase angle increases from 0^0 to -90^0. As can be seen from the figure, the tip of the current vector traces the path of a semicircle with its diameter along the +ve V axis but now on to its left side.
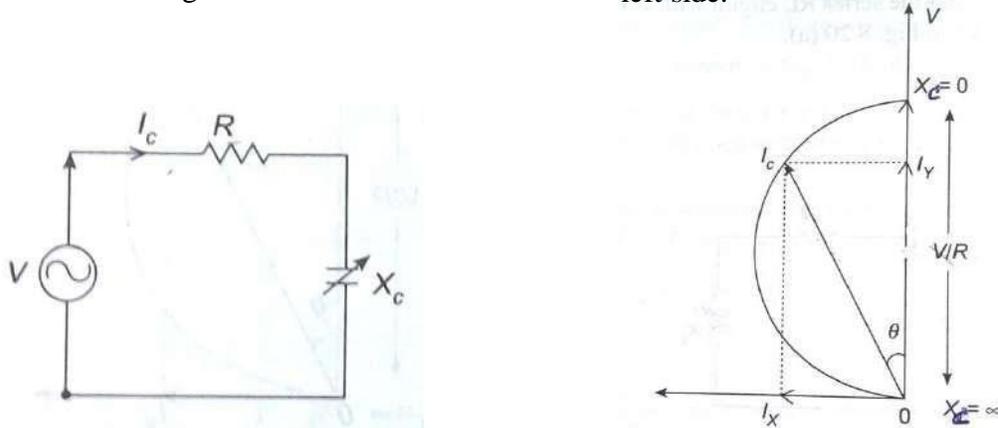
Fig(a): Series RC circuit with varying X_c Fig(b): Locus of current vector I_c

with variation of X_c Parallel LC circuits:

Parallel LC circuit along with its internal resistances as shown in the figures below is considered here for drawing the locus diagrams. As can be seen, there are two branch currents I_C and I_L along with the total current I. Locus diagrams of the current I_L or I_C (depending on which arm is varied)and the total current I are drawn by varying R_L, R_C , X_L and X_C one by one.

Varying X_L:
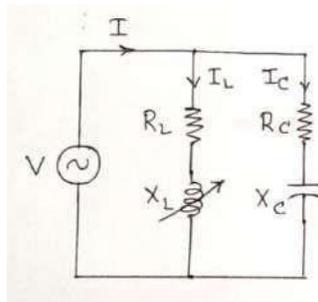
Fig(a): parallel LC circuit with Internal Resistances R_L and R_c in series with L (Variable) and C(fixed) respectively.

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\Theta_C = \tan^{-1} (X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L. It's amplitude is maximum and equal to V/R_L when X_L is zero and it is in phase with the applied voltage V. When X_L is increased from zero to infinity it's amplitude decreases to zero and phase will be lagging the voltage by 90^0. In between, the phase angle will be lagging the voltage V by an angle $\Theta_L = \tan^{-1} (X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with X_L varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L. We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semicircle $I_C I B_T$ shown in the figure in dotted lines is the locus of the total current vector OI.


Fig (b): Locus of current vector I in Parallel LC circuit when X_L is varied from 0 to ∞

Varying X_C:
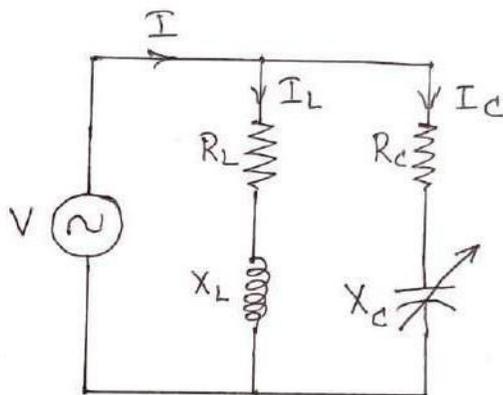
Fig.(a) parallel LC circuit with Internal Resistances R_L and R_c in series with L (fixed) and C(Variable) respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\Theta_L = \tan^{-1}(X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C. It's amplitude is maximum and equal to V/R_C when X_C is zero and it is in phase with the applied voltage V. When X_C is increased from zero to infinity it's amplitudedecreases to zero and phase will be leading the voltage by 90^0. In between, the phase angle will be leading the voltage V by an angle $\Theta_C = \tan^{-1}(X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/R_C as shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with X_C varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L . We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

**Fig(b) : Locus of current vector I in Parallel LC circuit when X_c is varied from 0 to ∞**

Varying R_L:

The current I_C through the capacitor is constant since R_C and C are fixed and it leads the voltage vector OV by an angle $\Theta_C = \tan^{-1} (X_C/R_C)$ as shown in the figure (b). The current I_L through the inductance is the vector OI_L. It's amplitude is maximum and equal to V/X_L when R_L is zero. Its phase will be lagging the voltage by 90^0. When R_L is increased from zero to infinity it's amplitude decreases to zero and it is in phase with the applied voltage V. In between, the phase angle will be lagging the voltage V by an angle $\Theta_L = \tan^{-1} (X_L/R_L)$. The locus of the current vector I_L is a semicircle with a diameter of length equal to V/R_L. Note that this is the same locus what we got earlier for the series RL circuit with R varying except that here V is shown horizontally.
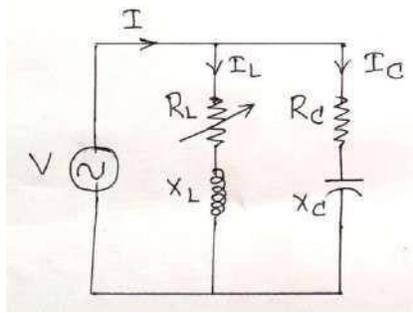
Fig.(a) parallel LC circuit with Internal Resistances R_L (Variable) and R_c (fixed) in series with L and C respectively.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L. We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_L)at the tip of the other vector (we will take constant amplitude vector I_C)and then join the start of fixed vector I_C to the end of varying vector I_L. Using this principle we can get the locus of the total current vector OI by shifting the I_L semicircle starting point O to the end of current vector OI_C keeping the two diameters parallel. The resulting semicircle I_CIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

Fig(b) : Locus of current vector I in Parallel LC circuit when R_L is varied from 0 to ∞

Varying R_C:
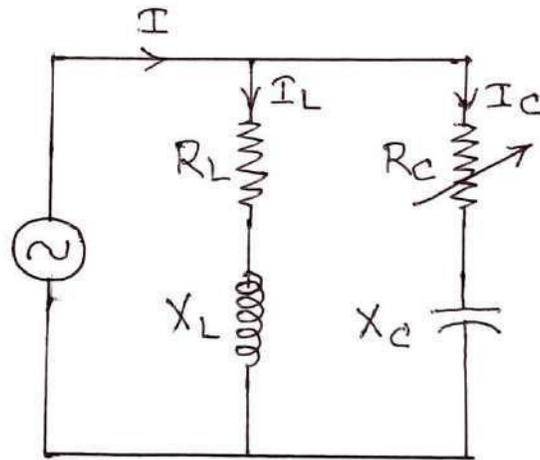
Fig.(a) parallel LC circuit with Internal Resistances R_L (fixed) and R_c (Variable) in series withL and C respectively.

The current I_L through the inductor is constant since R_L and L are fixed and it lags the voltage vector OV by an angle $\Theta_L = \tan^{-1} (X_L/R_L)$ as shown in the figure (b). The current I_C through the capacitance is the vector OI_C. It's amplitude is maximum and equal to V/X_C when R_C is zero and its phase will be leading the voltage by 90^0. When R_C is increased from zero to infinity it's amplitude decreases to zero and it will be in phase with the applied voltage V. In between, the phase angle will be leading the voltage V by an angle $\Theta_C = \tan^{-1} (X_C/R_C)$. The locus of the current vector I_C is a semicircle with a diameter of length equal to V/X_C as shown in the figure below. Note that this is the same locus what we got earlier for the series RC circuit with R varying except that here V is shown horizontally.

Now, to get the locus of the total current vector OI we have to add vectorially the currents I_C and I_L. We know that to get the sum of two vectors geometrically we have to place one of the vectors staring point (we will take varying amplitude vector I_C)at the tip of the other vector (we will take constant amplitude vector I_L) and then join the start of the fixed vector I_L to the end of varying vector I_C. Using this principle we can get the locus of the total current vector OI by shifting the I_C semicircle starting point O to the end of current vector OI_L keeping the two diameters parallel. The resulting semicircle I_LIB_T shown in the figure in dotted lines is the locus of the total current vector OI.

Fig(b) : Locus of current vector I in Parallel LC circuit when R_C is varied from 0 to ∞

Resonance :

Series RLC circuit:

The impedance of the series RLC circuit shown in the figure below and the current I through the circuit are given by :

$$Z = R + j\omega L + 1/j\omega C = R + j(\omega L - 1/\omega C)I = Vs/Z$$
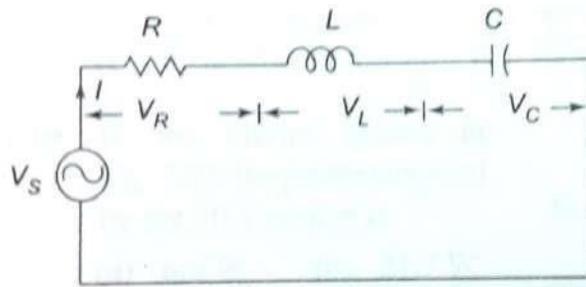
Fig: Series RLC circuit

The circuit is said to be in resonance when the Inductive reactance is equal to the Capacitive reactance. i.e. $X_L = X_C$ **or** $\omega L = 1/\omega C$. (i.e. Imaginary of the impedance is zero) The frequency

at which the resonance occurs is called resonant frequency. In the resonant condition when X_L = X_C they cancel with each other since they are in phase opposition(180^0 out of phase) and net impedance of the circuit is purely resistive. In this condition the magnitudes of voltages across the Capacitance and the Inductance are also equal to each other but again since they are of opposite polarity they cancel with each other and the entire applied voltage appears across the Resistance alone.

Solving for the resonant frequency from the above condition of Resonance : $\omega L = 1/\omega C$

$$2\pi f_r L = 1/2\pi f_r C$$

$$f_r^2 = 1/4\pi^2 LC \quad \text{and} \quad f_r = 1/2\pi\sqrt{LC}$$

In a series RLC circuit, resonance may be produced by varying L or C at a fixed frequency or by varying frequency at fixed L and C.

Reactances, Impedance and Resistance of a Series RLC circuit as a function of frequency:

From the expressions for the Inductive and capacitive reactances we can see that when the frequency is zero, capacitance acts as an open circuit and Inductance as a short circuit. Similarly when the frequency is infinity inductance acts as an open circuit and the capacitance acts as a short circuit. The variation of Inductive and capacitive reactances along with Resistance R and the Total Impedance are shown plotted in the figure below.

As can be seen, when the frequency increases from zero to ∞ Inductive reactance X_L (directly proportional to ω) increases from zero to ∞ and Capacitive reactance X_C (inversely proportional to ω) decreases from $-\infty$ to zero. Whereas, the Impedance decreases from ∞ to Pure Resistance R as the frequency increases from zero to f_r (as capacitive reactance reduces from $-\infty$ and becomes equal to Inductive reactance) and then increases from R to ∞ as the frequency increases from f_r to ∞ (as inductive reactance increases from its value at resonant frequency to ∞)
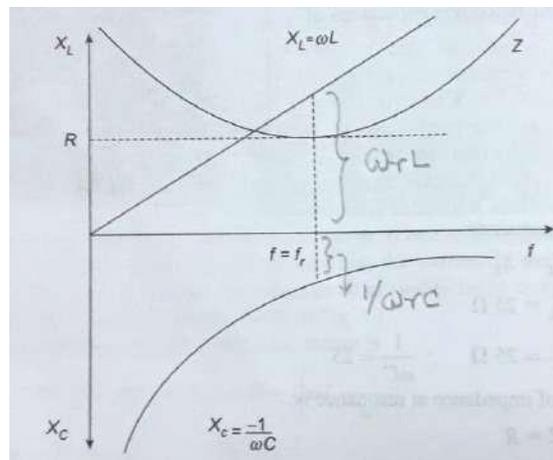

Fig : Reactance and Impedance plots of a Series RLC circuit

Phase angle of a Series RLC circuit as a function of frequency:
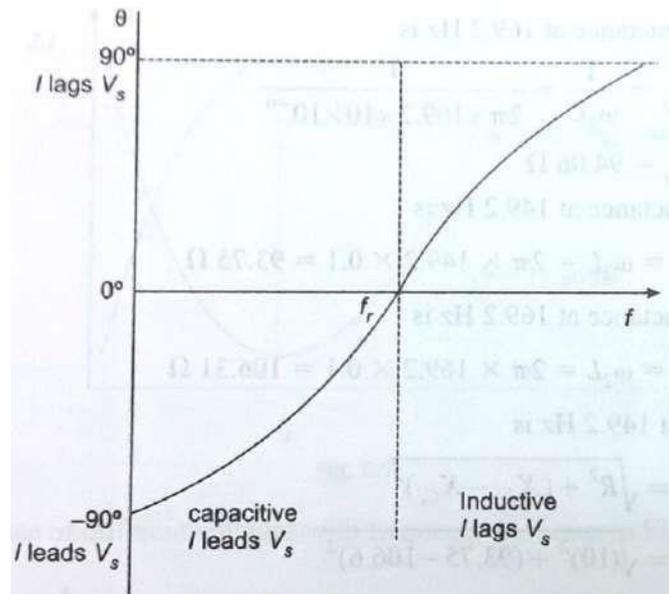

Fig : Phase plot of a Series RLC circuit

The following points can be seen from the Phase angle plot shown in the figure above:

- At frequencies below the resonant frequency capacitive reactance is higher than the inductive reactance and hence the phase angle of the current leads the voltage.
- As frequency increases from zero to f_r the phase angle changes from -90^0 to zero.
- At frequencies above the resonant frequency inductive reactance is higher than the capacitive reactance and hence the phase angle of the current lags the voltage.
- As frequency increases from f_r and approaches ∞, the phase angle increases from zero and approaches 90^0

Band width of a Series RLC circuit:

The band width of a circuit is defined as the Range of frequencies between which the output power is half of or 3 db less than the output power at the resonant frequency.These frequencies are called the cutoff frequencies, 3db points or half power points. But when we consider the output voltage or current, the range of frequencies between which the output voltage or current falls to 0.707 times of the value at the resonant frequency is called the Bandwidth BW. This is because voltage/current are related to power by a factor of $\sqrt 2$ and when we are consider $\sqrt 2$ times less it becomes 0.707. But still these frequencies are called

as cutoff frequencies, 3db points or half power points. The lower end frequency is called *lower cutoff frequency* and the higher end frequency is called upper *cutoff frequency.*
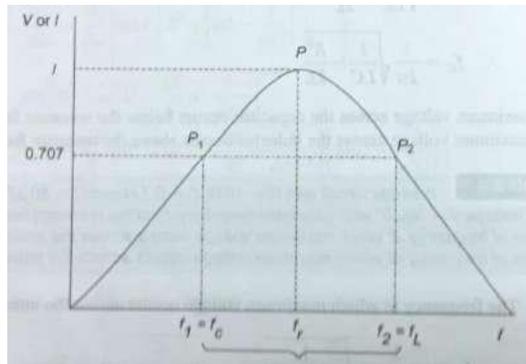

Fig: Plot showing the cutoff frequencies and Bandwidth of a series RLC circuit

Derivation of an expression for the BW of a series RLC circuit:
We know that $BW = f_2 - f_1$ Hz

If the current at points P_1 and P_2 are 0.707 $(1/\sqrt{2})$ times that of I_{max} (current at the resonant frequency) then the Impedance of the circuit at points P_1 and P_2 is $\sqrt{2} R$ (i.e. $\sqrt{2}$ times the impedance at f_r)

But Impedance at point P_1 is given by: $Z = \sqrt{R^2 + (1/\omega_1 C - \omega_1 L)^2}$ and equating this to $\sqrt{2} R$
we get : $(1/\omega_1 C) - \omega_1 L = R$ ----------------(1)

Similarly Impedance at point P_2 is given by: $Z = \sqrt{R^2 + (\omega_2 L - 1/\omega_2 C)^2}$ and equating this to $\sqrt{2} R$ we get: $\omega_2 L - (1/\omega_2 C) = R$ ------ (2)

Equating the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L = \omega_2 L - 1/\omega_2 C$$

Rearranging we get $L(\omega_1 + \omega_2) = 1/C[(\omega_1 + \omega_2)/\omega_1 \omega_2]$ i.e $\omega_1 \omega_2 = 1/LC$

But we already know that for a series RLC circuit the resonant frequency is given by $\omega_r^2 = 1/LC$

Therefore: $\omega_1 \omega_2 = \omega_r^2$ ---- (3) and $1/C = \omega_r^2 L$ ------ (4)

Next adding the above equations (1) and (2) we get:

$$1/\omega_1 C - \omega_1 L + \omega_2 L - 1/\omega_2 C = 2R$$

$$(\omega_2 - \omega_1)L + (1/\omega_1 C - 1/\omega_2 C) = 2R$$

$$(\omega_2 - \omega_1)L + 1/C[(\omega_2 - \omega_1)/\omega_1 \omega_2] = 2R \qquad ------ \qquad (5)$$

Using the values of $\omega_1 \omega_2$ and $1/C$ from equations (3) and (4) above into equation (5) above we get: $(\omega_2 - \omega_1)L + \omega_r^2 L[(\omega_2 - \omega_1)/\omega_r^2] = 2R$

i.e. $2L(\omega_2 - \omega_1) = 2R$ i.e. $(\omega_2 - \omega_1) = R/L$ and $(f_2 - f_1) = R/2\pi L$ --------- (6)

Or finally Band width $\qquad\qquad \mathbf{BW = R/2\pi L}$ --- (7)

Since f_r lies in the centre of the lower and upper cutoff frequencies f_1 and f_2 using the above equation (6) we can get:

$$f_1 \ = \ f_r - R/4\pi L \qquad \text{------ (8)}$$
$$f_2 \ = \ f_r + R/4\pi L \qquad \text{------ (9)}$$

Further by dividing the equation (6) above by f_r on both sides we get another **import nt** relation : $(f_2 - f_1)/f_r \ = \ R/2\pi f_r L$ **or** $BW/f_r \ = \ R/2\pi f_r L$ -------------- (10)

Here an important property of a coil i.e. **Q factor** or **figure of merit** is defined as the ratio of the reactance to the resistance of a coil.

$$Q \ = \ 2\pi f_r L / R \text{ -------------------------- (11)}$$

Now using the relation (11) we can rewrite the relation (10) as

$$Q \ = \ f_r / BW \text{ ------------------------------ (12)}$$

Quality factor of a series RLC circuit:

The quality factor of a series RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance / Average power at Resonance

Reactive power in Inductor at resonance $= I^2 X_L$
Reactive power in Capacitor at resonance $= I^2 X_C$
Average power at Resonance $=$ $I^2 R$

Here the power is expressed in the form $I^2 X$ (not as V^2/X) since I is common through R.L and C in the series RLC circuit and it gets cancelled during the simplification.

Therefore $\mathbf{Q \ = \ I^2 X_L / I^2 R \ = \ I^2 X_C / I^2 R}$

i.e. $\mathbf{Q \ = \ X_L / R \ = \ \omega_r L / R}$-------------------------------- (1)

Or $\mathbf{Q \ = \ X_C / R \ = \ 1/\omega_r RC}$-------------------------------- (2)

From these two relations we can also define Q factor as :

Q = Inductive (or Capacitive) reactance at resonance / Resistance

Substituting the value of $\mathbf{\omega_r \ = \ 1/\sqrt{LC}}$ in the expressions (1) or (2) for **Q** above we can get the value of **Q** in terms of **R, L,C** as below.

$$Q \ = \ (1/\sqrt{LC}) \, L/R \ = \ (1/R)(\sqrt{L/C})$$

Selectivity:

Selectivity of a series **RLC** circuit indicates how well the given circuit responds to a given resonant frequency and how well it rejects all other frequencies. i.e. the selectivity is directly proportional to **Q** factor. A circuit with a good selectivity (or a high **Q** factor) will have maximumgain at the resonant frequency and will have minimum gain at other frequencies .i.e. it will havevery low band width. This is illustrated in the figure below.
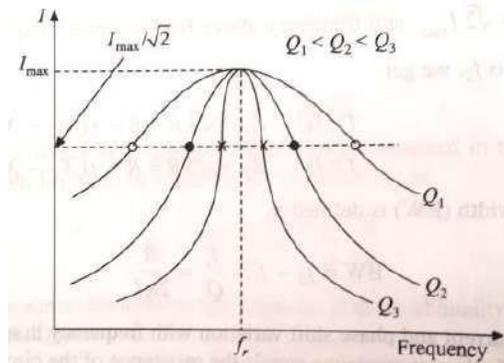
Fig: Effect of quality factor on bandwidthVoltage Magnification at resonance:

At resonance the voltages across the Inductance and capacitance are much larger than the applied voltage in a series RLC circuit and this is called voltage magnification at Resonance. The voltage magnification is equal to the **Q** factor of the circuit. This is proven below.

If we take the voltage applied to the circuit as **V** and the current through the circuit at resonance as **I** then

The voltage across the inductance **L** is: $V_L = IX_L = (V/R) \omega_r L$

andThe voltage across the capacitance **C** is: $V_C = IX_C = V/R \omega_r C$

But we know that the **Q** of a series RLC circuit $= \omega_r L/R = 1/R \omega_r C$

Using these relations in the expressions for V_L and V_C given above we

get$V_L = VQ$ and $V_C = VQ$

The ratio of voltage across the Inductor or capacitor at resonance to the applied voltage in a series RLC circuit is called Voltage magnification and is given by

Magnification = Q = V_L/V or

V_c/ VImportant points In Series RLC circuit at

resonant frequency :

- The impedance of the circuit becomes purely resistive and minimum i.e **Z = R**
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is **Q** times higher than the voltage across the resistor

Bandwidth and Q factor of a Parallel RLC circuit:

Parallel RLC circuit is shown in the figure below. For finding out the **BW** and **Q** factor of a parallel RLC circuit, since it is easier we will work with Admittance , Conductance and Susceptance instead of Impedance ,Resistance and Reactance like in series RLC circuit.
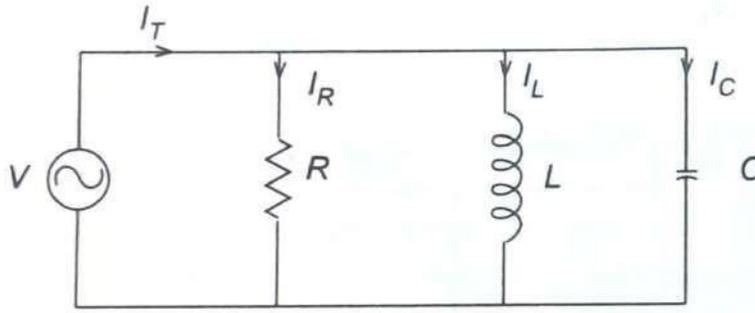
Fig: Parallel RLC circuit

Then we have the relation: $\qquad Y = 1/Z = 1/R + 1/j\omega L + j\omega C = 1/R + j(\omega C - 1/\omega L)$

For the parallel RLC circuit also, at resonance, the imaginary part of the Admittance is zero and hence the frequency at which resonance occurs is given by: $\quad \omega_r C - 1/\omega_r L = 0$. From this we get : $\qquad\qquad\qquad \omega_r C = 1/\omega_r L$ **and** $\omega_r = 1/\sqrt{LC}$
which is the same value for ω_r as what we got for the series RLC circuit.

At resonance when the imaginary part of the admittance is zero the ***admittance*** becomes ***minimum*** .(i.e ***Impedance*** becomes ***maximum*** as against Impedance becoming minimum in series RLC circuit) i.e. Current becomes minimum in the parallel RLC circuit at resonance (as against current becoming maximum in series RLC circuit) and increases on either side of the resonant frequency as shown in the figure below.
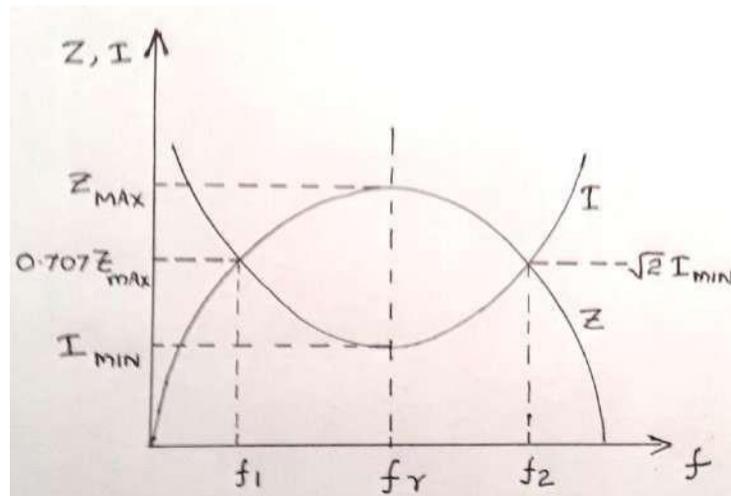
Fig: Variation of Impedance and Current with frequency in a Parallel RLC circuit

Here also the BW of the circuit is given by $BW = f_2-f_1$ where f_2 and f_1 are still called the upper and lower cut off frequencies but they are 3db higher cutoff frequencies since we notice that at these cutoff frequencies the amplitude of the current is $\sqrt{2}$ times higher than that of the amplitude of current at the resonant frequency.

The BW is computed here also on the same lines as we did for the series RLC circuit:

If the current at points P_1 and P_2 is $\sqrt{2}$ (3db) times higher than that of I_{min}(current at the resonant frequency) then the admittance of the circuit at points P_1 and P_2 is also $\sqrt{2}$ times higher than the admittance at f_r)

But amplitude of admittance at point P_1 is given by: $Y = \sqrt{1/R^2 + (1/\omega_1 L - \omega_1 C)^2}$ and equating this to $\sqrt{2}/R$ we get

$1/\omega_1 L - \omega_1 C \quad = \quad 1/R$ -------------- (1)

Similarly amplitude of admittance at point P_2 is given by: $Y = \sqrt{1/R^2 + (\omega_2 C - 1/\omega_2 L)^2}$ and equating this to $\sqrt{2}/R$ we get

$\omega_2 C - 1/\omega_2 L \quad = \quad 1/R$ -------------- (2)

Equating LHS of (1) and (2) and further simplifying we get

$$1/\omega_1 L - \omega_1 C = \omega_2 C - 1/\omega_2 L$$
$$1/\omega_1 L + 1/\omega_2 L = \omega_1 C + \omega_2 C$$
$$1/L \,[(\omega_1 + \omega_2)/\omega_1\omega_2] = (\omega_1 + \omega_2)C$$
$$1/L\,C = \omega_1\omega_2$$

Next adding the equations (1) and (2) above and further simplifying we get

$$1/\omega_1 L - \omega_1 C + \omega_2 C - 1/\omega_2 L = 2/R$$
$$(\omega_2 C - \omega_1 C) + (1/\omega_1 L - 1/\omega_2 L) =$$
$$2/R$$
$$(\omega_2 - \omega_1)C + 1/L\,[(\omega_2 - \omega_1)/\omega_1\omega_2] = 2/R$$

Substituting the value of $\omega_1\omega_2 = 1/LC$

$$(\omega_2 - \omega_1)C + LC/L\,[(\omega_2 - \omega_1)] =$$
$$2/R(\omega_2 - \omega_1)C + C\,[(\omega_2 - \omega_1)] =$$
$$2/R$$
$$2\,C\,[(\omega_2 - \omega_1)] = 2/R$$
$$Or\,[(\omega_2 - \omega_1)] =$$
$$1/RC$$

From which we get the band width $BW = f_2-f_1 = 1/2\pi\,RC$

Dividing both sides by f_r we get : $\quad (f_2-f_1)/f_r = 1/2\pi\,f_r\,RC \quad\quad (1)$

Quality factor of a Parallel RLC circuit:

The quality factor of a Parallel RLC circuit is defined as:

Q = Reactive power in Inductor (or Capacitor) at resonance / Average power at Resonance

Reactive power in Inductor at resonance = V^2/X_L
Reactive power in Capacitor at resonance = V^2/X_C
Average power at Resonance = V^2/R

Here the power is expressed in the form V^2/X (not as $I^2 X$ as in series circuit) since V is common across R,L and C in the parallel RLC circuit and it gets cancelled during the simplification. Therefore $Q = (V^2/X_L)/(V^2/R) = (V^2/X_C)/(V^2/R)$

i.e. $Q = R/X_L = R/\omega_r L$ ------------------------------------- (1)

Or $Q = R/X_C = \omega_r RC$ ------------------------------------- (2)

From these two relations we can also define **Q** factor as :

Q = Resistance /Inductive (or Capacitive) reactance at resonance

Substituting the value of $\omega_r = 1/\sqrt{LC}$ in the expressions (1) or (2) for **Q** above we can get the value of **Q** in terms of R, L,C as below.

Q = (1/√LC) RC= R (√C/L)

Further using the relation $Q = \omega_r RC$ (equation 2 above) in the earlier equation (1) we got in BW viz. $(f_2-f_1)/f_r = 1/2\pi f_r RC$ we get : $(f_2-f_1)/f_r = 1/Q$ or $Q = f_r / (f_2-f_1) = f_r / BW$

i.e. In Parallel RLC circuit also the Q factor is inversely proportional to the BW.

Admittance, Conductance and Susceptance curves for a Parallel RLC circuit as a function offrequency :

- The effect of varying the frequency on the Admittance, Conductance and Susceptance of a parallel circuit is shown in the figure below.
- Inductive susceptance B_L is given by $B_L = -1/\omega L$. It is inversely proportional to the frequency ω and is shown in the in the fourth quadrant since it is negative.
- Capacitive susceptance B_C is given by $B_C = \omega C$. It is directly proportional to the frequency ω and is shown in the in the first quadrant as OP .It is positive and linear.
- Net susceptance $B = B_C - B_L$ and is represented by the curve **JK**. As can be seen it is zero at the resonant frequency f_r
- The conductance $G = 1/R$ and is constant
- The total admittance **Y** and the total current **I** are minimum at the resonant frequency as shown by the curve **VW**
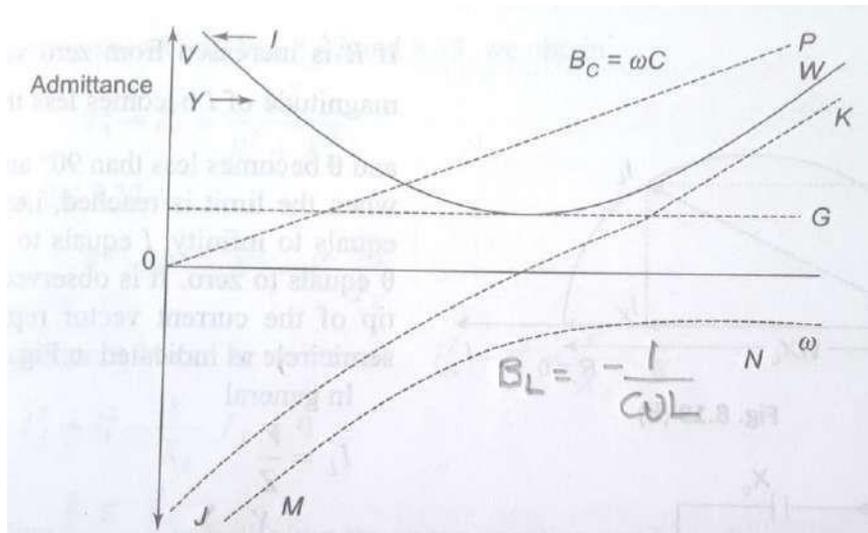
Fig: Conductance,Susceptance and Admittance plots of a Parallel RLC circuit

Current magnification in a Parallel RLC circuit:

Just as voltage magnification takes place across the capacitance and Inductance at the resonant frequency in a series RLC circuit , current magnification takes place in the currents through the capacitance and Inductance at the resonant frequency in a Parallel RLC circuit. This is shown below.

Voltage across the Resistance $= \mathbf{V} = \mathbf{IR}$

Current through the Inductance at resonance $\mathbf{I_L = V/ \omega_r \, L = IR / \omega_r \, L = I \, . \, R/ \omega_r \, L = I \, Q}$

Similarly

Current through the Capacitance at resonance $\mathbf{I_C = V/ (1/\omega_r \, C) = IR / (1/\omega_r \, C) = I(R \, \omega_r \, C) = I \, Q}$ From which we notice that the quality factor $\mathbf{Q = I_L / I \; or \; I_C / I}$ and that the current through theinductance and the capacitance increases by $\mathbf{Q}$ times that of the current through the resistor at resonance. .

Important points In Parallel RLC circuit at resonant frequency :

- The impedance of the circuit becomes resistive and maximum i.e **Z = R**
- The current in the circuit becomes minimum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The current through the Capacitor becomes equal and opposite to the current through the Inductor at resonance and is Q times higher than the current through the resistor

Magnetic Circuits:
Introduction to the Magnetic Field:

Magnetic fields are the fundamental medium through which energy is converted from one formto another in motors, generators and transformers. Four basic principles describe how magnetic fields are used in these devices.

1. A current-carrying conductor produces a magnetic field in the area around it.

Explained in Detail by Fleming's Right hand rule and Amperes Law.

2. A time varying magnetic flux induces a voltage in a coil of wire if it passes through that coil.
   (basis of Transformer action)

Explained in detail by the Faradays laws of Electromagnetic Induction.

3. A current carrying conductor in the presence of a magnetic field has a force induced in it (Basis of Motor action)
4. A moving wire in the presence of a magnetic field has a voltage induced in it (Basis of Generator action)

We will be studying in this unit the first two principles in detail and the other two principles in the next unit on DC machines.

Two basic laws governing the production of a magnetic field by a current carrying conductor : The direction of the magnetic field produced by a current carrying conductor is given by the *Flemings Right hand rule* and its' amplitude is given by the *Ampere's Law*.

Flemings right hand rule: Hold the conductor carrying the current in your right hand such that the Thumb points along the wire in the direction of the flow of current, then the fingers will encircle the wire along the lines of the Magnetic force.
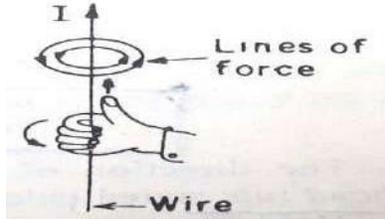
Ampere's Law : *The line integral of the magnetic field intensity H around a closed magnetic path is equal to the total current enclosed by the path.*

This is the basic law which gives the relationship between the Magnetic field Intensity H and the current I and is mathematically expressed as

$$\oint H.dl = I_{net}$$

where **H** is the magnetic field intensity produced by the current **I_net** and **dl** is a differential element of length along the path of integration. **H** is measured in *Ampere-turns per meter*.

Important parameters and their relation in magnetic circuits :

- Consider a current carrying conductor wrapped around a ferromagnetic core as shown in the figure below .
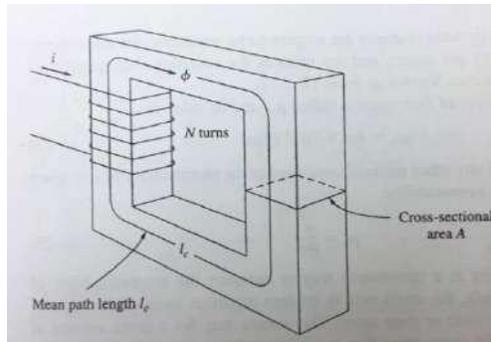
- Applying Ampere's law, the total amount of magnetic field induced will be proportional to the amount of current flowing through the conductor wound with N turns around the ferromagnetic material as shown. Since the core is made of ferromagnetic material, it is assumed that a majority of the magnetic field will be confined to the core.

- The path of integration in this case as per the Ampere's law is the mean path length of the core, l_c. The current passing within the path of integration I_{net} is then **Ni**, since the coil of wire cuts the path of integration **N** times while carrying the current **i**. Hence Ampere's Law becomes :   *Hl$_c$ = Ni*

  Therefore   *H = Ni/l$_c$*

- In this sense, **H** (Ampere turns per meter) is known as the effort required to induce a magnetic field. The strength of the magnetic field flux produced in the core also depends on the material of the core. Thus: ***B = µH*** where

  B = magnetic flux density [webers per square meter, or Tesla (T)]

  µ= magnetic permeability of material (Henrys per meter)

  H = magnetic field intensity (ampere-turns per meter)

- The constant **µ** may be further expanded to include ***relative permeability*** which can be defined as below:

  µ$_r$ =   µ /µ$_o$

  where **µ$_o$** = permeability of free space (equal to that of air)

- Hence the permeability value is a combination of the relative permeability and the permeability of free space. The value of relative permeability is dependent upon the type of material used. The higher the amount permeability, the higher the amount of flux induced in the core. Relative permeability is a convenient way to compare the magnetizability of materials.

- Also, because the permeability of iron is so much higher than that of air, the majority of the flux in an iron core remains inside the core instead of travelling through the surrounding air, which has lower permeability. The small leakage flux that does leave the iron core is important in determining the flux linkages between coils and the self-inductances of coils in transformers and motors.

- In a core such as shown in the figure above

$$B = µH = µ \, Ni/l_c$$

  Now, to measure the total flux flowing in the ferromagnetic core, consideration has to be made in terms of its cross sectional area (CSA). Therefore:

  $$\Phi = \int B.\, dA \text{ where: } A = \text{cross sectional area throughout the core.}$$

  Assuming that the flux density in the ferromagnetic core is constant throughout hence the equation simplifies to: **Φ = B.A**

  Taking the previous expression for **B**   we get **Φ = *µ NiA/l$_c$***

Electrical analogy of magnetic circuits:

The flow of magnetic flux induced in the ferromagnetic core is analogous to the flow of electric current in an electrical circuit hence the name magnetic circuit.

The analogy is as follows:
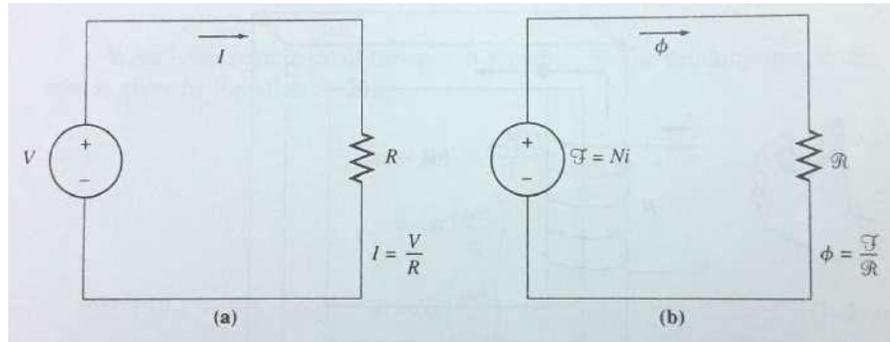
(a) Electric Circuit   (b) Electrical Analogy of Magnetic Circuit

- Referring to the magnetic circuit analogy, **F** is denoted as **magnetomotive force** (mmf) which is similar to Electromotive force in an electrical circuit (emf). Therefore, we can say that **F** is the force which pushes magnetic flux around a ferromagnetic core with a value of **Ni** (refer to ampere's law). Hence **F** is measured in ampere turns. Hence the magnetic circuit equivalent equation is as shown:

$$F = \emptyset.R \text{ (similar to V=IR)}$$

We already have the relation $\Phi = \mu\, NiA/l$ and using this we get $R = F / \Phi = Ni/ \Phi$

$$R = Ni /(\mu\, NiA/l) = l/ \mu\, A$$

- The polarity of the mmf will determine the direction of flux. To easily determine the direction of flux, the *'right hand curl'* rule is applied:

When the direction of the curled fingers indicates the direction of current flow the resulting thumb direction will show the magnetic flux flow.

- The element of **R** in the magnetic circuit analogy is similar in concept to the electrical resistance. It is basically the measure of material resistance to the flow of magnetic flux. **Reluctance** in this analogy obeys the rule of electrical resistance (Series and Parallel Rules). Reluctance is measured in *Ampere-turns per weber*.

- The inverse of electrical resistance is conductance which is a measure of conductivity of a material. Similarly the inverse of reluctance is known as **permeance P** which represents the degree to which the material permits the flow of magnetic flux.

- By using the magnetic circuit approach, calculations related to the magnetic field in a ferromagnetic material are simplified but with a little inaccuracy.

Equivalent Reluctance of a series Magnetic circuit : $R_{eqseries} = R_1 + R_2 + R_3 + ...$

Equivalent Reluctance of a Parallel Magnetic circuit: $1/R_{eqparallel} = 1/R_1 + 1/R_2 + 1/R_3 + ...$

Electromagnetic Induction and Faraday's law – Induced Voltage from a Time-ChangingMagnetic Field:

Faraday's Law:

Whenever a varying magnetic flux passes through a turn of a coil of wire, voltage will be induced in the turn of the wire that is directly proportional to the rate of change of the flux linkage with the turn of the coil of wire.

$$e_{ind} \ \alpha \ -d\emptyset/dt$$
$$e_{ind} = -k.d\emptyset/dt$$

The negative sign in the equation above is in accordance to **Lenz' Law** which states:

The direction of the induced voltage in the turn of the coil is such that if the coil is short circuited, it would produce a current that would cause a flux which opposes the original change of flux.

And **k** is the constant of proportionality whose value depends on the system of units chosen. In the SI system of units **k=1** and the above equation becomes:

$$e_{ind} = - d\emptyset/dt$$

Normally a coil is used with several turns and if there are N number of turns in the coil with the same amount of flux flowing through it then: $e_{ind} = - N \, d\emptyset/dt$

Change in the flux linkage N∅ of a coil can be obtained in two ways:

1. Coil remains stationary and flux changes with time (Due to AC current like in Transformers and this is called Statically induced e.m.f)
2. Magnetic flux remains constant and stationary in space, but the coil moves relative to the magnetic field so as to create a change in the flux linkage of the coil (Like in Rotating machines and this is a called Dynamically induced e.m.f.

Self inductance:

From the Faradays laws of Electromagnetic Induction we have seen that an e.m.f will beinduced in a conductor when a time varying flux is linked with a conductor and the amplitude of the induced e.m.f is proportional to the rate of change of the varying flux.

If the time varying flux is produced by a coil of **N** turns then the coil itself links with the time varying flux produced by itself and an emf will be induced in the same coil. This is called self inductance .

The flux **∅** produced by a coil of N turns links with its own N turns of the coil and hence the total flux linkage is equal to **N∅ = (μ N² A / l) I** *[using the expression Φ = μ NiA/l we already*

*develop*ed] Thus we see that the total magnetic flux produced by a coil of N turns and linked with itself is proportional to the current flowing through the coil i.e.

$$N\emptyset \text{ a } I \text{ or } N\emptyset = L\,I$$

From the Faradays law of electromagnetic Induction, the self induced e.m.f for this coil of N turns is given by:

$$e_{ind} = -N\,d\emptyset/dt = -L\,dI/dt$$

The constant of proportionality **L** is called the self Inductance of the coil or simply Inductance and its value is given by $L = (\mu\,N^2\,A\,/\,l)$. If the radius of the coil is **r** then:

$$L = (\mu\,N^2\,\pi r^2\,/\,l)\,i$$

From the above two equations we can see that Self Inductance of a coil can be defined as the flux produced per unit current i.e *Weber/Ampere* (equation1) or the induced emf per unit rate of change of current i.e *Volt-sec/Ampere* (equation 2)

The unit of Inductance is named after Joseph Henry as **Henry** and is given to these two combinations as :

$$1H = 1WbA^{-1} = 1VsA^{-1}$$

Self Inductance of a coil is defined as one Henry if an induced emf of one volt is generated when the current in the coil changes at the rate of one Ampere per second.

Henry is relatively a very big unit of Inductance and we normally use Inductors of the size of mH(10^{-3} H) or μH (10^{-3}H)

Mutual inductance and Coefficient of coupling:

In the case of Self Inductance an emf is induced in the same coil which produces the varying magnetic field. The same phenomenon of Induction will be extended to a separate second coilif it is located in the vicinity of the varying magnetic field produced by the first coil. Faradays law of electromagnetic Induction is equally applicable to the second coil also. A current flowing in one coil establishes a magnetic flux about that coil and also about a second coil nearby but of course with a lesser intensity. The time-varying flux produced by the first coil and surrounding the second coil produces a voltage across the terminals of the second coil. This voltage is proportional to the time rate of change of the current flowing through the first coil.

Figure (*a*) shows a simple model of two coils L_1 and L_2, sufficiently close together that the flux produced by a current $i_1(t)$ flowing through L_1 establishes an open-circuit voltage $v_2(t)$ across the terminals of L_2. *Mutual inductance,M21*, is defined such that

$$v_2(t) = M_{21}di_1(t)/dt \text{ ------------------ [1]}$$
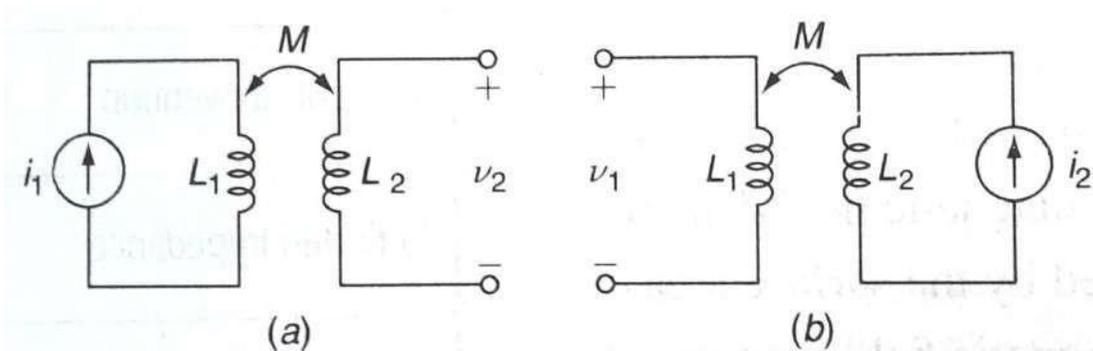
**Figure (*a*) A current *i*1 through *L*1 produces an open-circuit voltage
*v*2across *L*2. (*b*) A current**
*i*2 through *L*2 produces an open-circuit voltage *v*1 across *L*1.

The order of the subscripts on M_{21} indicates that a voltage response is produced at L_2 by a current source at L_1. If the system is reversed, as indicated
in fig.(*b*) then we have

$$v_1(t) = M_{12}di_2(t)/dt \text{------------------ [2]}$$

It can be proved that the two mutual inductances M_{12} and M_{21} are equal and thus, $M_{12} = M_{21} = M$. The existence of mutual coupling between two coils is indicated by a double-headed arrow, as shown in Fig. (*a*)and (*b*)

Mutual inductance is measured in Henrys and, like resistance, inductance, and capacitance, is a positive quantity. The voltage $M\ di/dt$, however, may appear as either a positive or a negative quantity depending on whether the current is increasing or decreasing at a particular instant of time.

Coefficient of coupling k **:** Is given by the relation $\mathbf{M = k\sqrt{L_1\,L_2}}$ and its value lies between 0 and 1. It can assume the maximum value of 1 when the two coils are wound on the same core such that flux produced by one coil completely links with the other coil. This is possible in well designed cores with high permeability. Transformers are designed to achieve a coefficient of coupling of 1.

Dot Convention:

The polarity of the voltage induced in a coil depends on the sense of winding of the coil. In the case of Mutual inductance it is indicated by use of a method called "***dot convention***". The dot convention makes use of a large dot placed at one end of each of the two coils which are mutually coupled. Sign of the mutual voltage is determined as follows:

> A current entering the dotted terminal of one coil produces an open circuit voltage with a positive voltage reference at the dotted terminal of the second coil.

Thus in Fig(*a*) i_1 enters the dotted terminal of L_1, v_2 is sensed positively at the dotted terminal of L_2, and $v_2 = M\ di_1/dt$.

It may not be always possible to select voltages or currents throughout a circuit so that the passive sign convention is everywhere satisfied; the same situation arises with mutual coupling. For example, it may be more convenient to represent v_2 by a positive voltage reference at the undotted terminal, as shown in Fig (b). Then $v_2 = -M\, di_1/dt$. Currents also may not always enter the dotted terminal as indicated by Fig (c) and (d). Then we note that:

A current entering the undotted terminal of one coil provides a voltage that is positively sensed at the undotted terminal of the second coil.
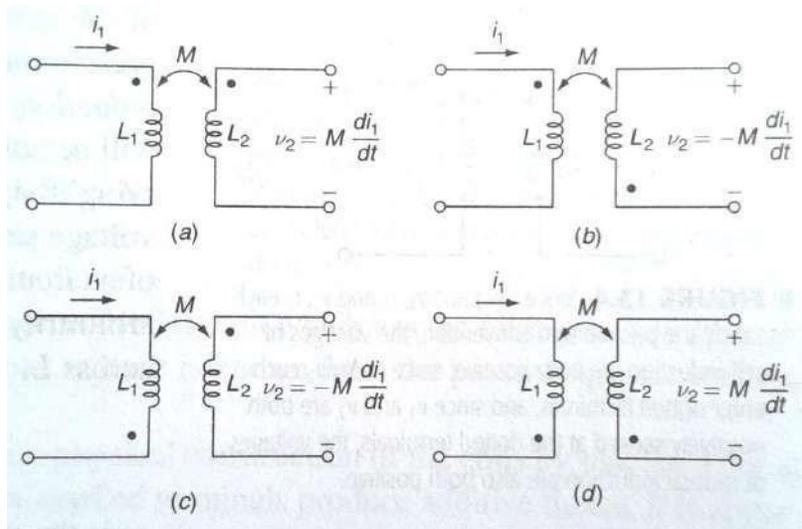
Figure : (a) and (b) Current entering the dotted terminal of one coil produces a voltage that is sensed positively at the dotted terminal of the second coil. **(c) and (d)** Current entering the undotted terminal of one coil produces a voltage that is sensed positively at the undotted terminal of the second coil.

Important Concepts and formulae:

Resonance and Series RLC circuit:

$$\omega_r^2 = \omega_1\omega_2 = 1/LC \therefore \omega_r = \sqrt{\omega_1\omega_2} = 1/\sqrt{LC}$$

$$BW = R/2\pi L$$

$$Q = \omega_r L / R = 1/\omega_r RC \text{ and in terms of R,L and C} = (1/R)(\sqrt{L/C})$$

$Q = f_r / BW$ **i.e.** inversely proportional to the BW

Voltage magnification Magnification = $Q = V_L/V$ or V_C / V

Important points In Series RLC circuit at resonant frequency :

- The impedance of the circuit becomes purely resistive and minimum i.e **Z = R**
- The current in the circuit becomes maximum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The voltage across the Capacitor becomes equal to the voltage across the Inductor at resonance and is **Q** times higher than the voltage across the resistor

Resonance and Parallel RLC circuit:

$$\omega_r^2 = \omega_1\omega_2 = 1/LC \therefore \omega_r = \sqrt{\omega_1\omega_2} = 1/\sqrt{LC} \text{ same as in series RLC circuit}$$

BW = $1/2\pi$ RC

$Q = R /\omega_r L = \omega_r RC$ **and in terms of R, L and C** $= R(\sqrt{C/L})$ [Inverse of what we got in Series RLC circuit]
$Q = f_r / BW$ In Parallel RLC also   inversely proportional to the BW

Current Magnification = $Q = I_L/I$ or I_C / I

Important points In Parallel RLC circuit at resonant frequency :

- The impedance of the circuit becomes resistive and maximum i.e **Z = R**
- The current in the circuit becomes minimum
- The magnitudes of the capacitive Reactance and Inductive Reactance become equal
- The current through the Capacitor becomes equal and opposite to the current through the Inductor at resonance and is Q times higher than the current through the resistor

Magnetic circuits :

Ampere's Law: $\oint H.\,dl = I_{net}$ and in the case of a simple closed magneticpath of a ferromagnetic material it simplifies to $Hl = Ni$ $or\ H = Ni/l$

Magnetic flux density: $B = \mu H$
Magnetic field intensity: $H = Ni/l$
Total magnetic flux intensity: $Ø = BA = \mu HA = \mu\ Ni\ A\ /\ l$
Reluctance of the magnetic circuit: $R = mmf/Flux = Ni/\ Ø = l/\mu A$

Faradays law of electromagnetic Induction:

Self induced e.m.f of a coil of N turns is given by: $e_{ind} = - N\ dØ/dt = -L\ dI/dt$ where L is theinductance of the coil of N turns with radius r and given by $L = (\mu\ N^2\ \pi r^2\ /\ l)\ i$
Equivalent Reluctance of a series Magnetic circuit: $R_{eqseries} = R_1 + R_2 + R_3 + ...$
Equivalent Reluctance of a Parallel Magnetic circuit: $1/R_{eqparallel} = 1/R_1 + 1/R_2 + 1/R_3 +$
..Coefficient of coupling k Is given by the relation: $M = k\sqrt{L_1\ L_2}$

UNIT-V

NETWORK THEOREMS

- ➢ Thevenin's Theorem
- ➢ Norton's Theorem
- ➢ Maximum Power Transfer Theorem
- ➢ Superposition Theorem
- ➢ Reciprocity Theorem
- ➢ Tellegen's Theorem
- ➢ Substitution Theorem
- ➢ Compensation Theorem
- ➢ Millman's Theorem

INTRODUCTION:

Any complicated network i.e. several sources, multiple resistors are present if the single element response is desired then use the network theorems. Network theorems are also can be termed as network reduction techniques. Each and every theorem got its importance of solving network. Let us see some important theorems with DC and AC excitation with detailed procedures.

Thevenin's Theorem and Norton's theorem (Introduction) :

Thevenin's Theorem and Norton's theorem are two important theorems in solving Network problems having many active and passive elements. Using these theorems the networks can be reduced to simple equivalent circuits with one active source and one element. In circuit analysis many a times the current through a branch is required to be found when it's value is changed with all other element values remaining same. In such cases finding out every time the branch current using the conventional mesh and node analysis methods is quite awkward and time consuming. But with the simple equivalent circuits (with one active source and one element) obtained using these two theorems the calculations become very simple. Thevenin's and Norton's theorems are dual theorems.

Thevenin's Theorem Statement :

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance (Impedance).The equivalent voltage source V_{Th} is the open circuit voltage looking into the terminals(with concerned branch element removed) and the equivalent resistance R_{Th} while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.
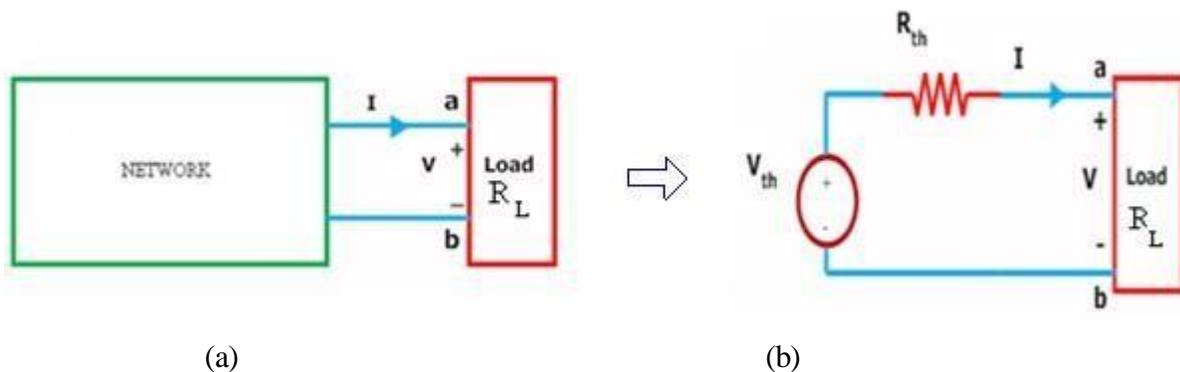
(a)   (b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance R_L connected across the terminals **'a & b'** and figure (b) shows the **Thevenin equivalent circuit** with V_{Th} connected across R_{Th} & R_L .

Main steps to find out V_{Th} and R_{Th} :

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

2. Open circuit voltage V_{OC} across these two terminals is found out using the conventional network mesh/node analysis methods and this would be V_{Th} .

3. **Thevenin resistance R_{Th}** is found out by the method depending upon whether the network contains dependent sources or not.

   a. With dependent sources: $R_{Th} = V_{oc} / I_{sc}$

   b. Without dependent sources : R_{Th} = **Equivalent resistance looking into the concerned terminals** with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with V_{Th} in series with R_{Th} and the concerned branch resistance **(or)** load resistance across the load terminals(A&B) as shown in below fig.

Thevenin's equivalent circuit
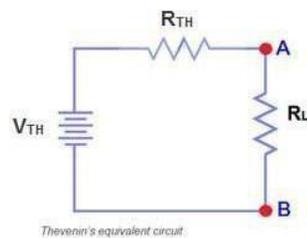
Example: Find V_{TH}, R_{TH} and the load current and load voltage flowing through R_L resistor as shown in fig. by using Thevenin's Theorem?
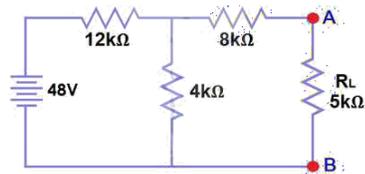
Fig.(a)

Solution:

The resistance R_L is removed and the terminals of the resistance R_L are marked as **A & B** as shown in the fig. (1)
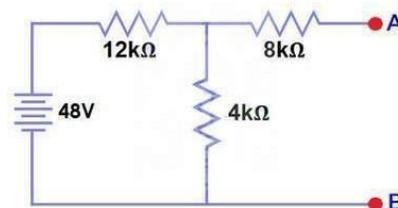
Fig.(1)

Calculate / measure the Open Circuit Voltage. This is the Thevenin Voltage (V$_{TH}$). We have already removed the load resistor from fig.(a), so the circuit became an open circuit as shown in fig (1). Now we have to calculate the Thevenin's Voltage. Since 3mA Current flows in both 12kΩ and 4kΩ resistors as this is a series circuit because current will not flow in the 8kΩ resistor as it is open. So 12V (3mA x 4kΩ) will appear across the 4kΩ resistor. We also know that current is not flowing through the 8kΩ resistor as it is open circuit, but the 8kΩ resistor is in parallel with 4k resistor. So the same voltage (i.e. 12V) will appear across the 8kΩ resistor as 4kΩ resistor. Therefore 12V will appear across the AB terminals. So, **V$_{TH}$ = 12V**
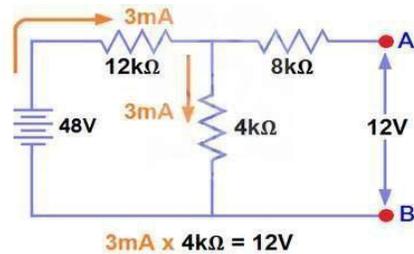
Fig(2)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) as shown in fig.(3)
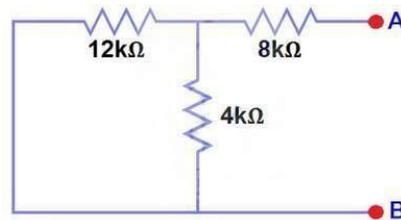
Fig(3)

Calculate /measure the Open Circuit Resistance. This is the Thevenin Resistance (R$_{TH}$)We have Reduced the 48V DC source to zero is equivalent to replace it with a short circuit as shown in figure (3) We can see that 8kΩ resistor is in series with a parallel connection of 4kΩ resistor and 12k Ω resistor. i.e.:

8kΩ + (4k Ω || 12kΩ) ….. (|| = in parallel with)

R$_{TH}$ = 8kΩ + [(4kΩ x 12kΩ) / (4kΩ + 12kΩ)]

R$_{TH}$ = 8kΩ + 3kΩ

R$_{TH}$ = 11kΩ
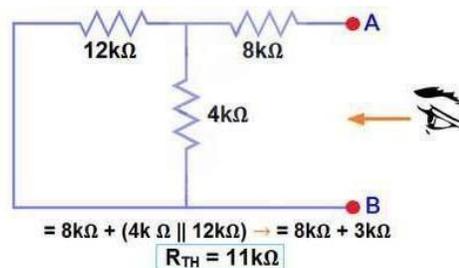
Fig(4)

Connect the R_{TH} in series with Voltage Source V_{TH} and re-connect the load resistor across the load terminals(A&B) as shown in fig (5) i.e. Thevenin circuit with load resistor. This is the Thevenin's equivalent circuit
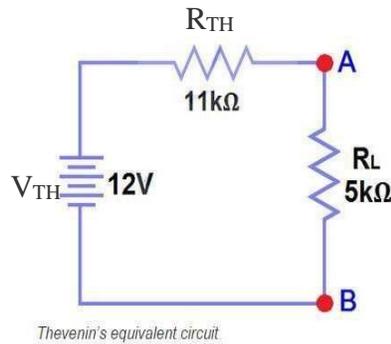
Thevenin's equivalent circuit

Fig(5)

Now apply Ohm's law and calculate the total load current from fig 5.

$I_L = V_{TH}/ (R_{TH} + R_L)= 12V / (11k\Omega + 5k\Omega) = 12/16k\Omega$

$I_L = 0.75mA$

And $V_L = I_L x R_L = 0.75mA \times 5k\Omega$

$V_L = 3.75V$

Norton's Theorem Statement :

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance),the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.
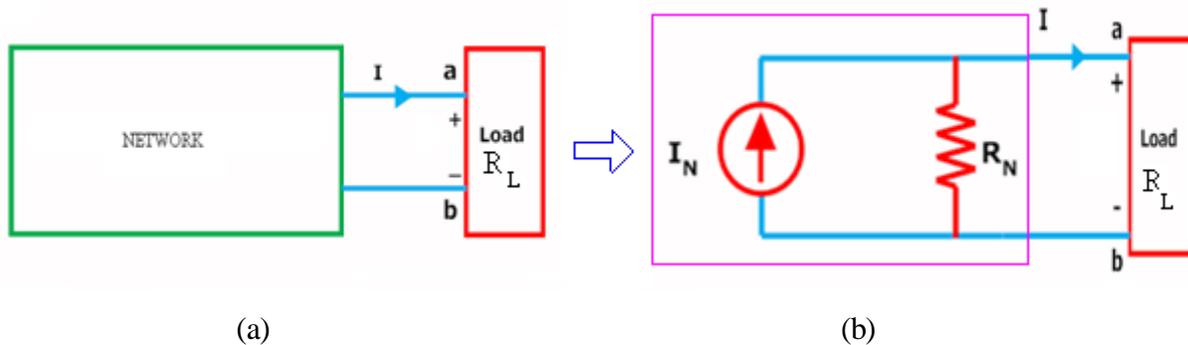
(a)   (b)

Figure (a) shows a simple block representation of a network with several active / passive elements with the load resistance **R_L** connected across the terminals '**a & b**' and figure (b) shows the **Norton equivalent circuit** with **I_N** connected across **R_N** & **R_L** .

Main steps to find out I_N and R_N:

1. The terminals of the branch/element through which the current is to be found out are marked as say **a & b** after removing the concerned branch/element.

2. Open circuit voltage V_{OC} across these two terminals and I_{SC} through these two terminals are found out using the conventional network mesh/node analysis methods and they are same as what we obtained in Thevenin's equivalent circuit.

3. Next **Norton resistance R_N** is found out depending upon whether the network contains dependent sources or not.

   a) With dependent sources: $R_N = V_{oc} / I_{sc}$

   b) Without dependent sources : R_N = Equivalent resistance looking into the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with I_N in parallel with R_N and the concerned branch resistance across the load terminals(A&B) as shown in below fig
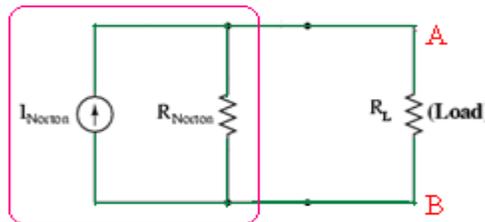
Example: Find the current through the resistance R_L (1.5 Ω) of the circuit shown in the figure (a) below using Norton's equivalent circuit.?
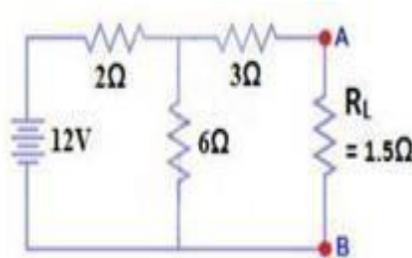
Fig(a)

Solution: To find out the Norton's equivalent ckt we have to find out $I_N = I_{sc}$,$R_N = V_{oc}/ I_{sc}$. Short the 1.5Ω load resistor as shown in (Fig 2), and Calculate / measure the Short Circuit Current. This is the Norton Current (I_N).
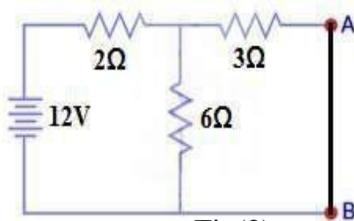
Fig(2)

We have shorted the AB terminals to determine the Norton current, I_N. The 6Ω and 3Ω are then in parallel and this parallel combination of 6Ω and 3Ω are then in series with 2Ω.So the Total Resistance of the circuit to the Source is:-

2Ω + (6Ω || 3Ω) ….. (|| = in parallel with)

$R_T = 2Ω + [(3Ω × 6Ω) / (3Ω + 6Ω)]$

$R_T = 2Ω + 2Ω$

$R_T = 4Ω$

$I_T = V / R_T$

$I_T = 12V / 4Ω = 3A..$

Now we have to find $I_{SC} = I_N$… Apply CDR… (Current Divider Rule)…

$I_{SC} = I_N = 3A × [(6Ω / (3Ω + 6Ω)] = 2A.$

$I_{SC} = I_N = 2A.$
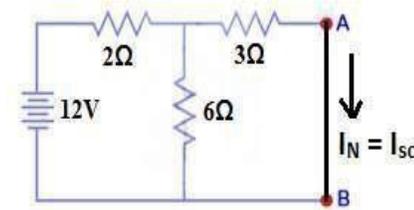
Fig(3)

All voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited) and Open Load Resistor. as shown in fig.(4)
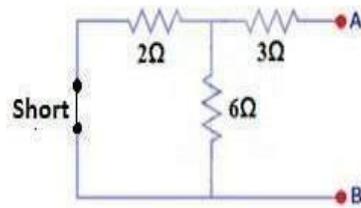
Fig(4)

Calculate /measure the Open Circuit Resistance. This is the Norton Resistance (R_N) We have Reduced the 12V DC source to zero is equivalent to replace it with a short circuit as shown in fig(4), We can see that 3Ω resistor is in series with a parallel combination of 6Ω resistor and 2Ω resistor. i.e.:

3Ω + (6Ω || 2Ω) ….. (|| = in parallel with)

$R_N = 3Ω + [(6Ω × 2Ω) / (6Ω + 2Ω)]$

$R_N = 3Ω + 1.5Ω$

$R_N = 4.5Ω$
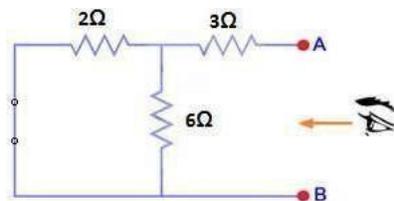
Fig(5)

Connect the R_N in Parallel with Current Source I_N and re-connect the load resistor. This is shown in fig (6) i.e. Norton Equivalent circuit with load resistor.
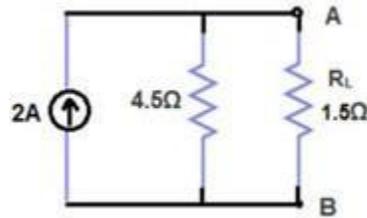
Fig(6)

Now apply the Ohm's Law and calculate the load current through Load resistance across the terminals A&B. Load Current through Load Resistor is

$I_L = I_N \times [R_N / (R_N + R_L)]$

$I_L = 2A \times (4.5\Omega / 4.5\Omega + 1.5k\Omega)$

$I_L = 1.5A$ $I_L = 1.5A$

Maximum Power Transfer Theorem:

In many practical situations, a circuit is designed to provide power to a load. While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for Efficiency and economic reasons, there are other applications in areas such as communications where it is desirable to maximize the power delivered to a load. electrical applications with electrical loads such as Loud speakers, antennas, motors etc. it would be required to find out the condition under which maximum power would be transferred from the circuit to the load.

Maximum Power Transfer Theorem Statement:

Any linear, bilateral two terminal network consisting of a resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.
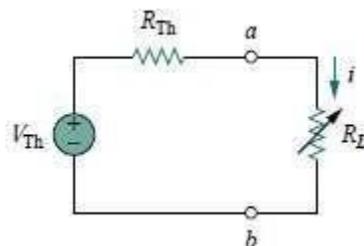
According to Maximum Power Transfer Theorem, for maximum power transfer from the network to the load resistance , R_L must be equal to the source resistance i.e. Network's Thevenin equivalent resistance R_{Th} . i.e. $R_L = R_{Th}$

The load current **I** in the circuit shown above is given by,

$$I = \frac{V_{TH}}{R_{TH}+R_L}$$

The power delivered by the circuit to the load:

$$P = I^2R = \frac{V_{TH}^2}{(R_{TH}+R_L)^2}R_L$$

The condition for maximum power transfer can be obtained by differentiating the above expression for power delivered with respect to the load resistance (Since we want to find out the value of **R$_L$** for maximum power transfer) and equating it to zero as :

$$\frac{\partial P}{\partial R_L} = 0 = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} - \frac{2V_{TH}^2}{(R_{TH}+R_L)^3}R_L = 0$$

Simplifying the above equation, we get:

$$(R_{TH} + R_L) - 2R_L = 0 \Longrightarrow R_L = R_{TH}$$

Under the condition of maximum power transfer, the power delivered to the load is given by :

$$P_{MAX} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_L = \frac{V_{TH}^2}{4R_L}$$

Under the condition of maximum power transfer, the efficiency **η** of the network is then given by:

$$P_{LOSS} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_{TH} = \frac{V_{TH}^2}{4R_L}$$

$$\eta = \frac{\text{output}}{\text{input}} = \frac{\frac{V_{TH}^2}{4R_L}}{(\frac{V_{TH}^2}{4R_L} + \frac{V_{TH}^2}{4R_L})} = 0.50$$

For maximum power transfer the load resistance should be equal to the Thevenin equivalent resistance (or Norton equivalent resistance) of the network to which it is connected . Under the condition of maximum power transfer the efficiency of the system is 50 %.

Example: Find the value of R$_L$ for maximum power transfer in the circuit of Fig. Find the maximum power.?
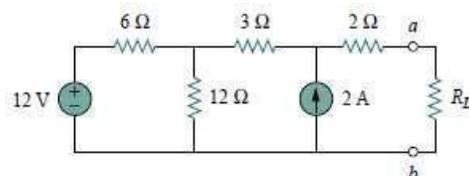
Solution: We need to find the Thevenin resistance R_{Th} and the Thevenin voltage V_{Th} across the terminals *a-b*. To get R_{Th}, we use the circuit in Fig. (a)

(a)   (b)
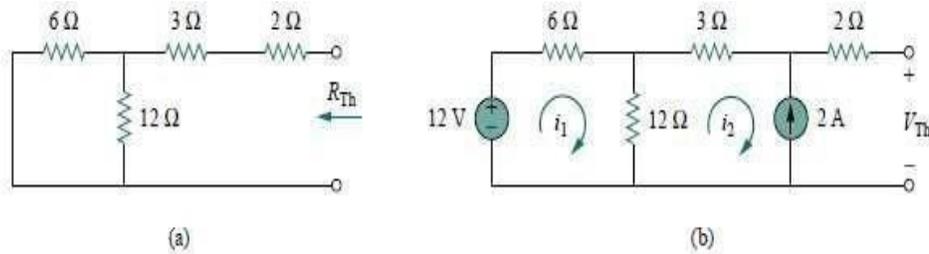
$R_{Th}= 2 + 3 + (6 // 12)=5+(\frac{6\times12}{6+12})=5+4=9\Omega$

To get V_{Th}, we consider the circuit in Fig.(b).Applying mesh analysis,

$-12 + 18i_1 - 12i_2 = 0,$

$i_2 = -2$ A,

Solving for i1, we get $i_1 = -2/3$.

Applying KVL around the outer loop to get V_{Th} across terminals *a-b*, we obtain,

$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0$

$V_{Th} = 22$ V

For maximum power transfer, $R_L = R_{Th} = 9\Omega$ and the maximum power is,

$$P_{MAX} = \frac{V_{TH}^2}{4R_L} = \frac{22\times22}{4\times9} = 13.44\text{W}$$

Superposition Theorem:

The principle of superposition helps us to analyze a linear circuit with more than one current or voltage sources sometimes it is easier to find out the voltage across or current in a branch of the circuit by considering the effect of one source at a time by replacing the other sources with their ideal internal resistances.

Superposition Theorem Statement:

Any linear, bilateral two terminal network consisting of more than one sources, The total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually while other sources are replaced by their ideal internal resistances. (i.e. Voltage sources by a short circuit and current sources by open circuit)

Steps to Apply Super position Principle:

1. Replace all independent sources with their internal resistances except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example: By Using the superposition theorem find I in the circuit shown in figure?
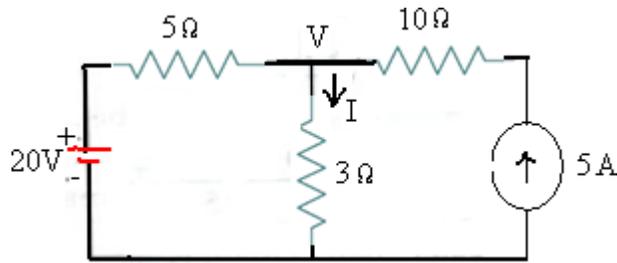
Fig.(a)

Solution: Applying the superposition theorem, the current I_2 in the resistance of 3 Ω due to the voltage source of 20V alone, with current source of 5A open circuited [as shown in the figure.1 below] is given by :
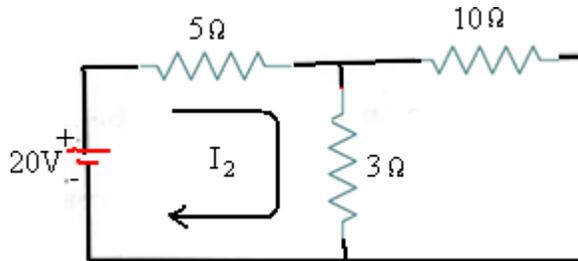
Fig1

$$I_2 = 20/(5+3) = 2.5A$$

Similarly the current I_5 in the resistance of 3 Ω due to the current source of 5A alone with voltage source of 20V short circuited [as shown in the figure.2 below] is given by :
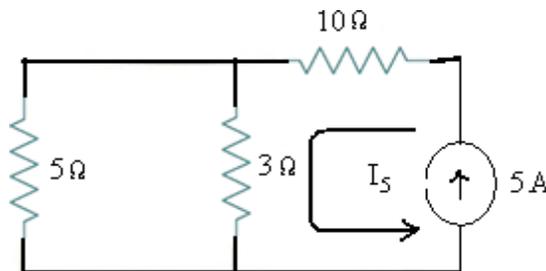
Fig.2

$$I_5 = 5 \times 5/(3+5) = 3.125 \text{ A}$$

The total current passing through the resistance of 3Ω is then $= I_2 + I_5 = 2.5 + 3.125 = \mathbf{5.625 \text{ A}}$

Let us verify the solution using the basic nodal analysis referring to the node marked with V in fig.(a).Then we get :

$$\frac{V - 20}{5} + \frac{V}{3} = 5$$

$$3V\text{-}60\text{+}5V\text{=}15\times 5$$

$$8V\text{-}60\text{=}75$$

$$8V\text{=}135$$

$$V\text{=}16.875$$

The current I passing through the resistance of 3Ω =V/3 = 16.875/3 = **5.625 A** .

Reciprocity theorem:

Under Basic Electrical Engineering In many electrical networks it is found that if the positions of voltage source and ammeter are interchanged, the reading of ammeter remains the same. Suppose a voltage source is connected to a passive network and an ammeter is connected to other part of the network to indicate the response. Now any one interchanges the positions of ammeter and voltage source that means he or she connects the voltage source at the part of the network where the ammeter was connected and connects ammeter to that part of the network where the voltage source was connected. The response of the ammeter means current through the ammeter would be the same in both the cases. This is where the property of reciprocity comes in the circuit. The particular circuit that has this reciprocal property, is called reciprocal circuit.

Reciprocity theorem Statement:

Any linear, bilateral two terminal network the ratio of excitation to response is constant even though the source is interchanged from input terminals to the output terminals.
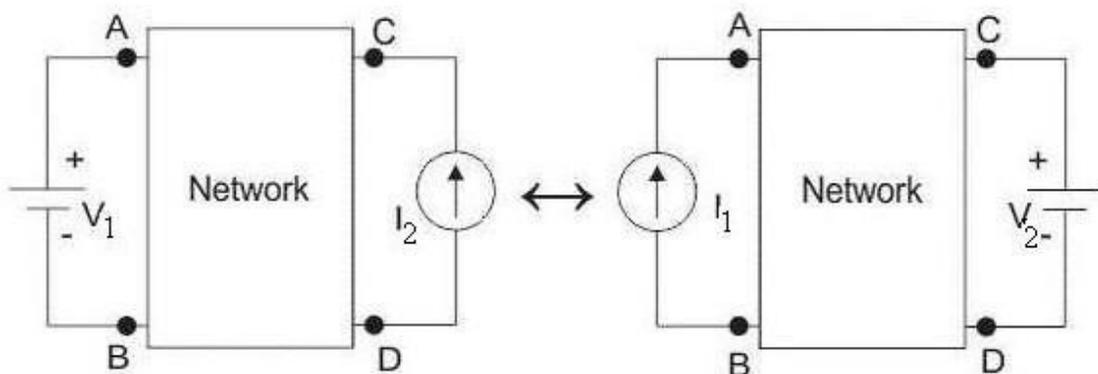
$$\frac{V_1}{I_2} = \frac{V_2}{I_1}$$

Steps For Solution Of a Network Utilizing Reciprocity Theorem:

1. The branches between which reciprocity is to be established to be selected first.

2. The current in the branch is obtained using conventional network analysis.

3. The voltage source is interchanged between the branches concerned.

4. The current in the branch where the voltage source was existing earlier is calculated.

   It may observe that the currents obtained in 2 & 4 are identical to each other.

Example: Verify the reciprocity theorem for the network shown in the figure (1).?
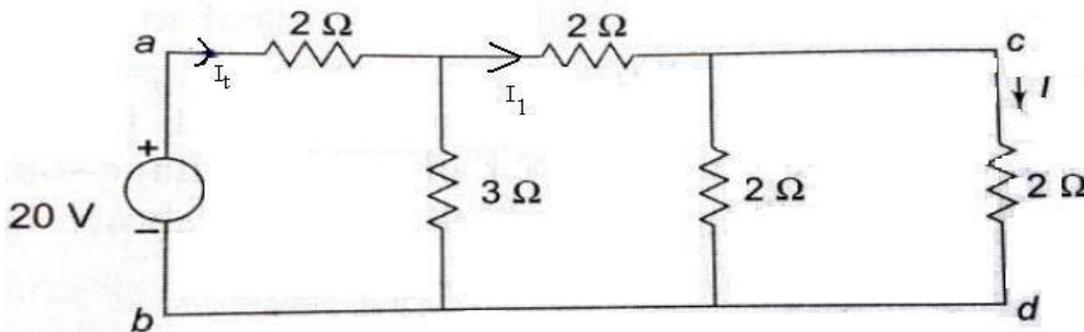
Fig.(1)

Solution: Total resistance in the circuit across the applied voltage of 20 V is

$R_{TH}=2 + [3||(2 +(2||2))]$

   $=2 +[3||3]$

   $=3.5\ \Omega$

The total current drawn by the circuit $I_T = \frac{V}{R_{TH}}$ = 20/3.5 = 5.71 A

The current **I** in the branch '**cd**' with 2 Ω resistance is find by using current division rule. For that first find I_1 current.

$$I_1=5.71\times\frac{3}{3+3}=2.855A$$

The current I in the 'cd' branch is

$$I=2.855\times\frac{2}{2+2}=1.427A$$

Now the source voltage and the response are interchanged between branches 'ab' and 'cd' as shown in the figure (2) below
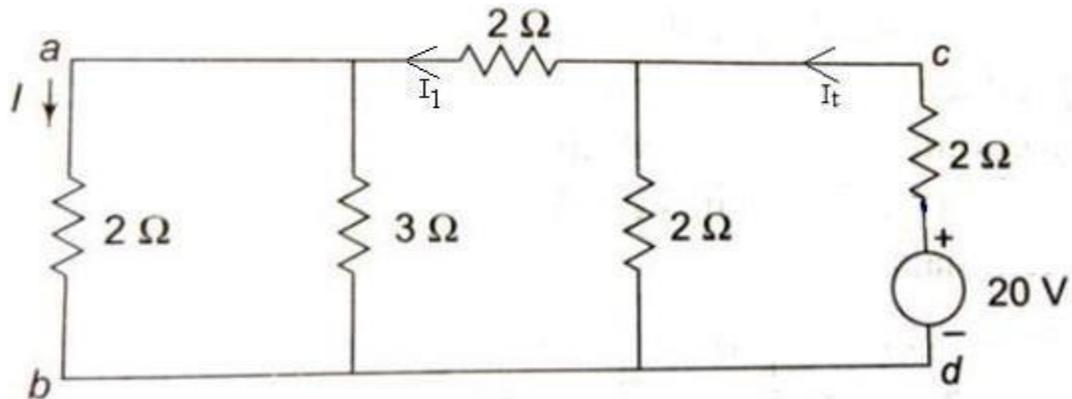
Fig.(2)

Total resistance in the circuit across the applied voltage of 20 V is

R_{TH}=2 + [2||(2 +(2||3))]

=2 +[2||3.2]

=3.23Ω

The total current drawn by the circuit $I_T = \dfrac{V}{R_{TH}}$=20/3.23=6.19A

The current **I** in the branch 'ab' with 2 Ω resistance is find by using current division rule. For that first find I_1 current.

$$I_1=6.19\times \dfrac{2}{3.2+2}=2.38A$$

The current I in the 'ab' branch is

$$I=2.38\times\dfrac{3}{3+2}=1.427A$$

The current in the branch 'ab' = **1.427 A** which is same as the current we got in branch 'cd' when the voltage was given from branch 'ab' . Thus the **reciprocity theorem** is verified.

Tellegen's Theorem:

This theorem is the one of the most general theorems in network analysis regardless to the type and nature, Tellegen's theorem is applicable to any network made up of lumped two terminal elements.

Tellegen's Theorem Statement:

In any linear, non-linear, passive, active, time variant or time invariant network the algebraic sum of power at any given instant is zero. Thus for K^{th} branch, this theorem states that

$$\sum_{K=1}^{n} v_K i_K = 0$$

Where, n=Being the number of branches,

V_K=Voltage drop in the branch,

I_K=Current drop in the branch.

It also evident that the sum of power delivered to the network is equal to the sum of power absorbed by the network elements.

Steps For Solution Of a Network Utilizing Tellegen's Theorem:

1. Find the branch voltage drops and corresponding branch currents using conventional analysis.
2. Summing all products of branch voltage and current.

Example: Verify the Tellegen's theorem for the given circuit.?
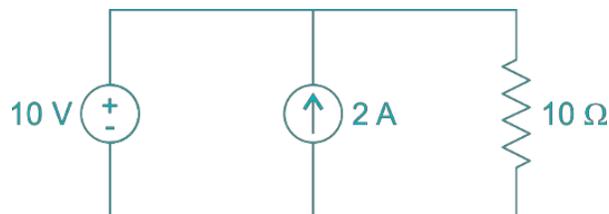
Solution: If current flows from + to − then treat it as power absorption.

If current flow from − to + then treat it as power delivering.
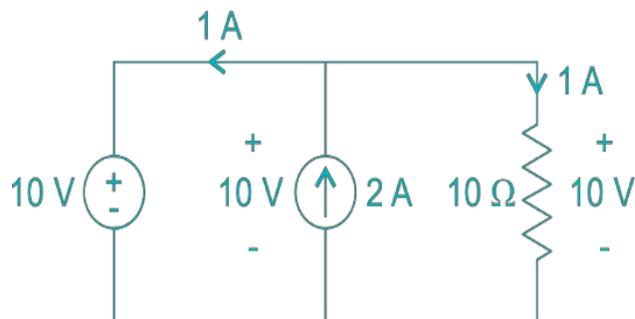
$\therefore P_{10V} = V.I = 10 \times 1 = 10$ watt ($P_{absorbed}$).

$P_{2A} = V.I = 10 \times 2 = 20$ watt $(P_{delivered})$

$P_{10\Omega} = I^2.R = 1^2 \times 10 = 10$ watt $(P_{absorbed})$

Pdelivered = Pabsorbed = 20 watt

Hence Tellegen's theorem is verified.

Substitution Theorem Statement:

The voltage across and the current through a Branch in a bilateral network is known, the branch can be replaced by any combination of elements in such a way that the same voltage will appear across and same current will pass through the chosen terminals. In other words for branch equivalence the terminal voltage and the current must be same.

This is illustrated with a simple circuit shown in the figure below.
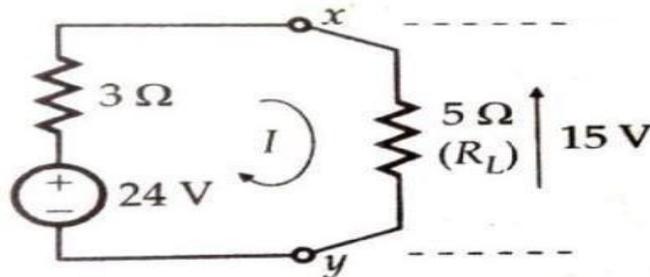
Figure: (a) A simple DC circuit to explain the substitution theorem.

In this circuit the load resistance R_L is the branch being considered for equivalence. The current **I** through the load resistance R_L = 24/(3+5) = 3 A.
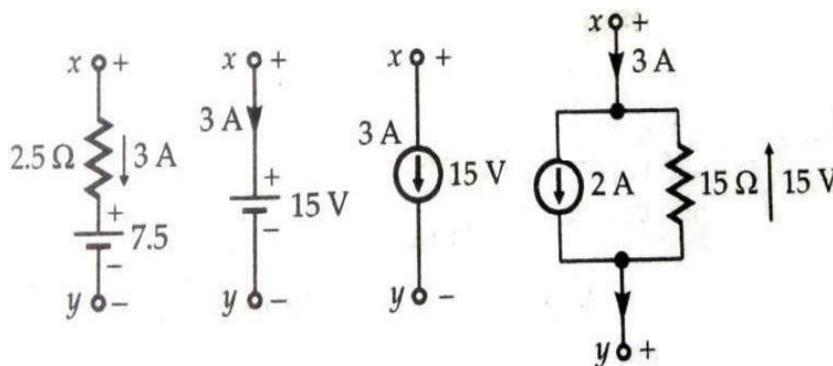
Figure: (b) Equivalent branches across terminals 'xy'

In figure (b) above several equivalents of branch **'xy'** are shown. It may be noted that in all the cases the terminal voltage across and the current through the equivalent branch are the same as that of the original branch R_L. It may also be observed that a known potential

difference and a current in a branch can be replaced by an ideal voltage source or an ideal current source respectively.

The limitation of this theorem is that it cannot be used to solve a network containing two or more sources that are not in series or parallel.

**Example: Using substitution theorem, draw equivalent branches for the branch 'a-b' of the network of Fig.(a)?**
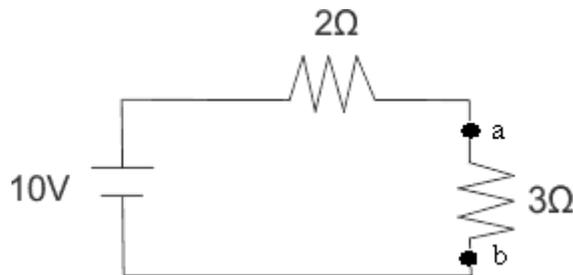
Fig.(a)

Solution: As per voltage division rule voltage across 3Ω and 2Ω resistance are

$$V_{3\Omega} = \frac{10 \times 3}{2+3} = 6V$$

$$V_{2\Omega} = \frac{10 \times 2}{2+3} = 4V$$

Current through the circuit is, $I = \frac{10}{2+3} = 2A$

If we replace the 3Ω resistance with a voltage source of 6 V as shown in fig (1), then
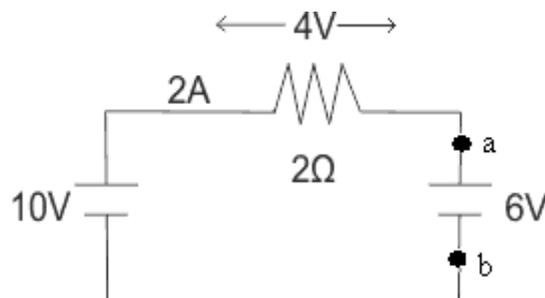
Fig.(1)

Then according to Ohm's Law the voltage across 2Ω resistance and current through the circuit is,

$$V_{2\Omega} = 10 - 6 = 4V$$

$$I = \frac{10-6}{2} = 2A$$

Alternately if we replace 3Ω resistance with a current source of 2A as shown in Fig(2),then
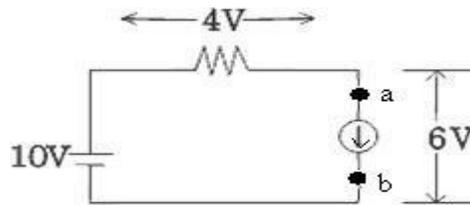
Fig.(2)

Voltage across 2Ω is $V_{2\Omega} = 10 - (3\times 2) = 4$ V and
Voltage across 2A current source is $V_{2A} = 10 - 4 = 6$ V.
The voltage across 2Ω resistance and current through the circuit is unaltered i.e. all initial condition of the circuit is intact.

Compensation Theorem:

It is one of the important theorems in Network Analysis , which finds its application mostly in calculating the sensitivity of electrical networks & bridges and solving electrical networks. In many circuits, after the circuit is analyzed, it is realized that only a small change need to be made to a component to get a desired result. In such a case we would normally have to recalculate. The compensation theorem allows us to compensate properly for such changes without sacrificing accuracy.

Compensation Theorem Statement:

In a linear, bilateral, time invariant network when the resistance (R) of an uncoupled branch, carrying a current (I), is changed by (ΔR). The currents in all the branches would change and can be obtained by assuming that an ideal voltage source of (VC) has been connected such that VC = I (ΔR) in series with (R + ΔR) when all other sources in the network are replaced by their internal resistances.

In Compensation Theorem, the source voltage (V_C) opposes the original current. In simple words compensation theorem can be stated as – the resistance of any network can be replaced by a voltage source, having the same voltage as the voltage drop across the resistance which is replaced.
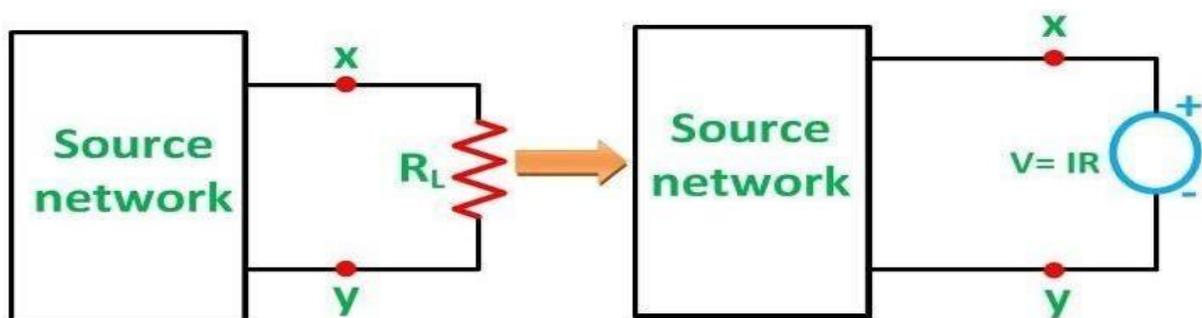
Let us assume a load R_L be connected to a DC source network whose Thevenin's equivalent gives V_0 as the Thevenin's voltage and R_{TH} as the Thevenin's resistance as shown in the figure below.
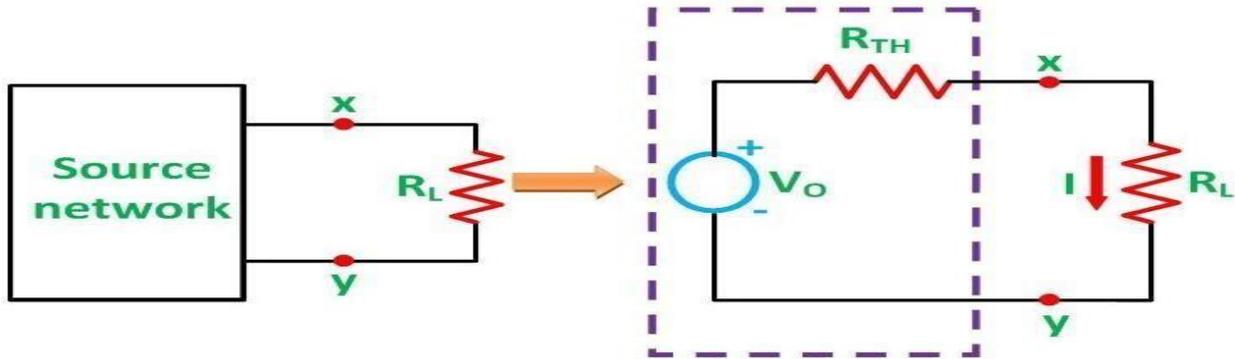
Here,

$$I = \frac{V_0}{R_{TH}+R_L} \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

Let the load resistance RL be changed to (RL + ΔRL). Since the rest of the circuit remains unchanged, the Thevenin's equivalent network remains the same as shown in the circuit diagram below

Thevenin's
equivalent source

Source network with source
replaced by internal resistance
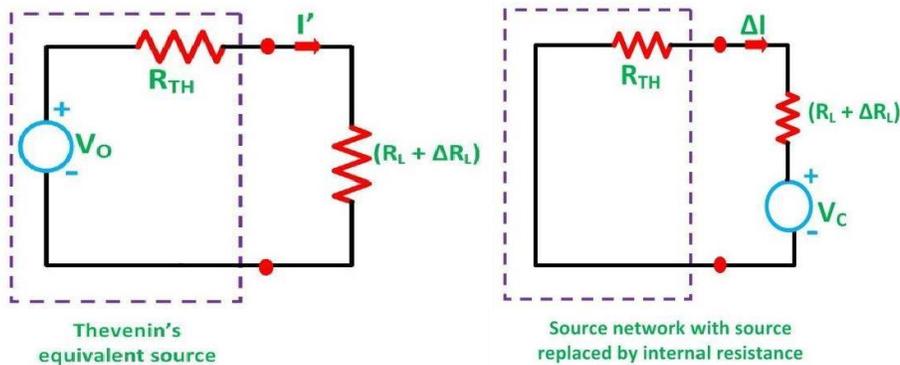
Here,

$$I' = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} \ldots \ldots \ldots \ldots \ldots \ldots (2)$$

The change of current being termed as ΔI Therefore,

$$\Delta I = I' - I \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Putting the value of I' and I from the equation (1) and (2) in the equation (3) we will get the following equation.

$$\Delta I = \frac{V_0}{R_{TH} + (R_L + \Delta R_L)} - \frac{V_0}{R_{TH} + R_L}$$

$$\Delta I = \frac{V_0\{(R_{TH}+R_L)-(R_{TH}+(R_L+\Delta R_L))\}}{(R_{TH}+(R_L+\Delta R_L))\times(R_{TH}+R_L)}$$

$$\Delta I = -[\frac{V_0}{R_{TH}+R_L}]\frac{R_{TH}}{R_{TH}+(R_L+\Delta R_L)} \quad (4)$$

Now, putting the value of I from the equation (1) in the equation (4), we will get the following equation.

$$I = -\frac{IR_{TH}}{R_{TH}+(R_L+\Delta R_L)} \quad (5)$$

As we know, $V_C = I \Delta R_L$ and is known as compensating voltage. Therefore, the equation (5) becomes.

$$\Delta I = \frac{-V_C}{R_{TH} + (R_L + \Delta R_L)}$$

Hence, Compensation Theorem tells that with the change of branch resistance, branch currents changes and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current, all other sources in the network being replaced by their internal resistances.

Example: Determine the current flowing through the ammeter having an internal resistance of 1 Ω connected in series with a 3 Ω resistor as shown in the fig (a).?
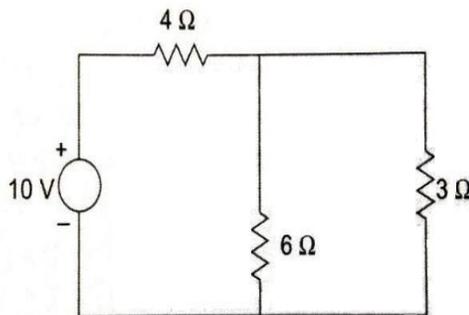
Fig.(a)

Solution:
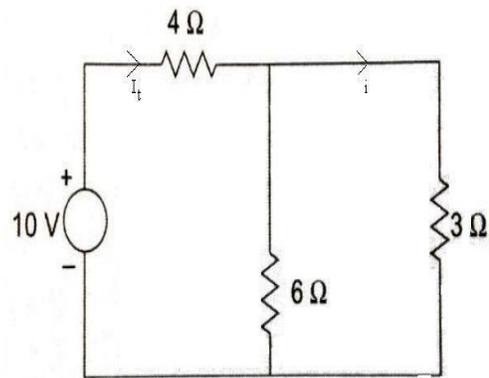
The current flowing through the 3 Ω branch i,

$$i = I_t [6/(6+3)]$$

$$I_t = \frac{10}{(4+(6\|3))}$$

$$I_t = \frac{10}{(4+2)}$$

$$I_t = 1.67A$$

$$i = 1.67[6/(6+3)]$$

$$i = 1.11A$$

Now when we connect the ammeter with an internal resistance of 1 Ω in the 3 Ω branch ,there is a change in resistance . This change in resistance causes currents in other branches as if a voltage source of voltage v is

V= i. R = 1.11x1 = 1.11v

1.11v is inserted in the 3 Ω branch as shown in the fig (1) below.
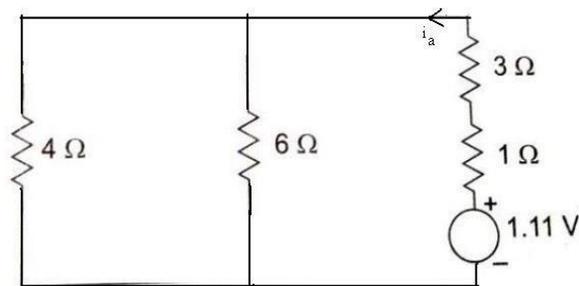
Fig.(1)

Current due to this additional source of 1.11 V in the 3 Ω branch i_a is,

$$i_a = \frac{1.11}{(1+3+(6\|4))}$$

$$i_a = \frac{1.11}{(1+3+2.4)}$$

$$i_a = 0.17A$$

This current flows in the opposite direction to that of the original current i through the 3 Ω branch(i.e. i_a is opposite to i)

Hence Ammeter reading = i_a-i=(1.11 – 0.17) = 0.94 A

Millman's Theorem:

Millman's Theorem is a theorem which helps in simplifying electrical networks with a bunch of parallel branches. The utility of this theorem that, any number of parallel voltage sources can be reduced to one equivalent one.

Millman's Theorem Statement:

The Millman's Theorem states that – when a number of voltage sources (V_1, V_2, V_3......... V_n) are in parallel having internal resistance (R_1, R_2, R_3.........................R_n) respectively, the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R.
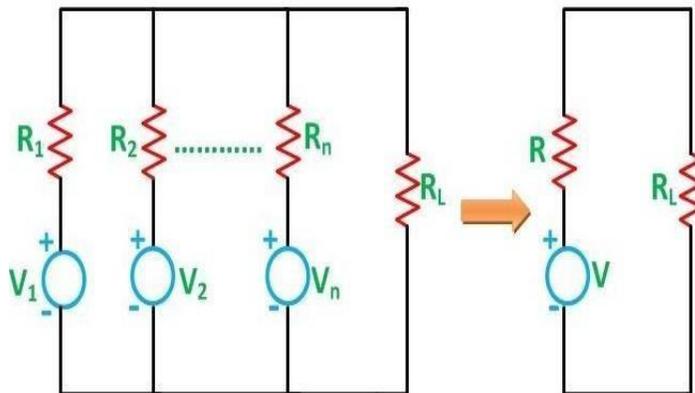
As per Millman's Theorem,

$$V = \frac{\pm V_1 G_1 \pm V_2 G_2 \pm \cdots \ldots \ldots \ldots \pm V_n G_n}{G_1 + G_2 + \cdots \ldots \ldots \ldots + G_n} \; ;$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots \ldots \ldots \ldots + G_n}$$

A DC network of numerous parallel voltage sources with internal resistances supplying power to a load resistance RL as shown in the figure below.
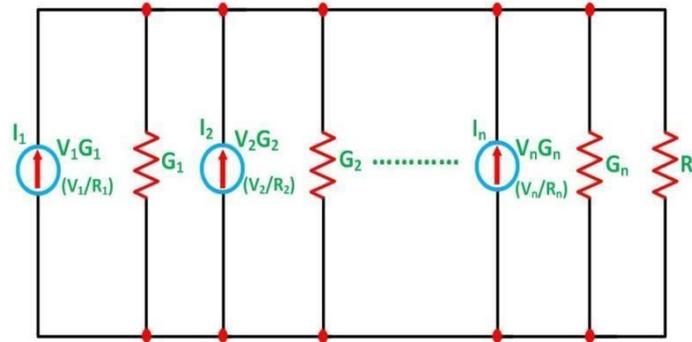
Let I represent the resultant current of the parallel current sources while G the equivalent conductance as shown in the figure below.
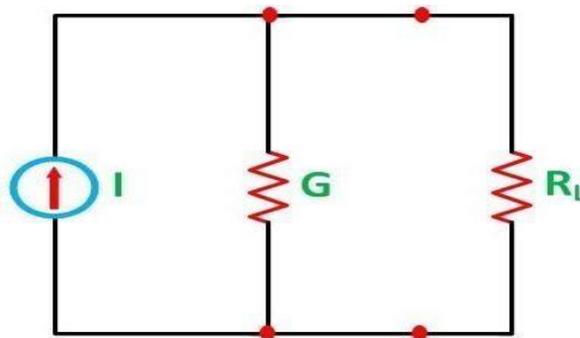
$$I = I_1 + I_2 + I_3 \text{................................};$$

$$G = G_1 + G_2 + G_3 \text{.............................(or)}$$

$$I = \frac{\pm I_1 R_1 \pm I_2 R_2 \pm \cdots \cdots \cdots \pm I_n R_n}{R_1 + R_2 + \cdots \cdots \cdots + R_n};$$

$$G = \frac{1}{R} = \frac{1}{R_1 + R_2 + \cdots \cdots \cdots \cdots + R_n}$$

The resulting current source is converted to an equivalent voltage source as shown in the fig.
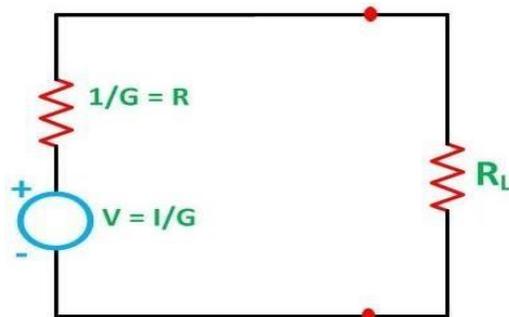
$$V = \frac{I}{G} = \frac{\pm I_1 \pm I_2 \pm \cdots \cdots \cdots \pm I_n}{G_1 + G_2 + \cdots \cdots \cdots + G_n}$$

$$R = \frac{1}{G} = \frac{1}{G_1 + G_2 + \cdots \cdots \cdots + G_n}$$

And as we know,$I = V/R$, and we can also write $R = 1/G$ as $G = 1/R$ So the equation can be written as,

$$V = \frac{\pm \dfrac{V_1}{R_1} \pm \dfrac{V_2}{R_2} \pm \cdots \cdots \cdots \pm \dfrac{V_n}{R_n}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \cdots \cdots \cdots + \dfrac{1}{R_n}}$$

Example: Find the value of current through R_L using Millman's theorem.?
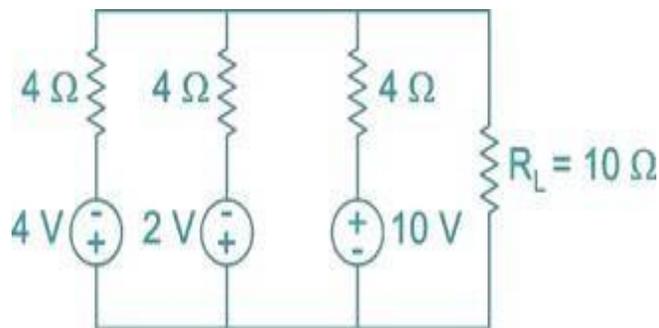
Solution:

Given $R_1 = R_2 = R_3 = 4$

$G = G_1 + G_2 + G_3$

$G = \dfrac{1}{4} + \dfrac{1}{4} + \dfrac{1}{4} = \dfrac{3}{4}$

$\therefore R = \dfrac{1}{G} = \dfrac{4}{3}\,\Omega$

$V = \dfrac{V_1 G_1 + V_2 G_2 + V_3 G_3}{G_1 + G_2 + G_3}$

$$V = \dfrac{(-4)\dfrac{1}{4} + (-2)\dfrac{1}{4} + (10)\dfrac{1}{4}}{\dfrac{3}{4}}$$

$$V = \dfrac{-4 - 2 + 10}{3}$$

$$V = \dfrac{4}{3}$$

So given circuit becomes,
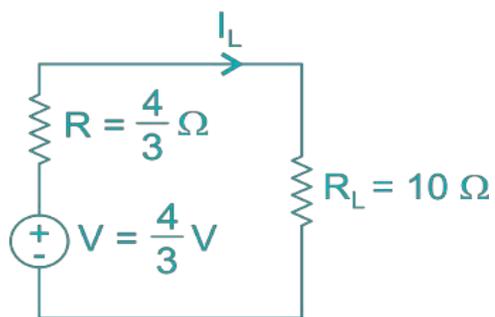
$$\therefore I_L = \dfrac{V}{R + R_L} = \dfrac{\dfrac{4}{3}}{\dfrac{4}{3} + 10} = \dfrac{4}{34} = 117.64 \text{mA}$$

The current flowing through R_L is **117.64mA**

**Problem :**Sketch the dotted equivalent circuit for the coupled coil shown in the fig. and find the equivalent inductive?
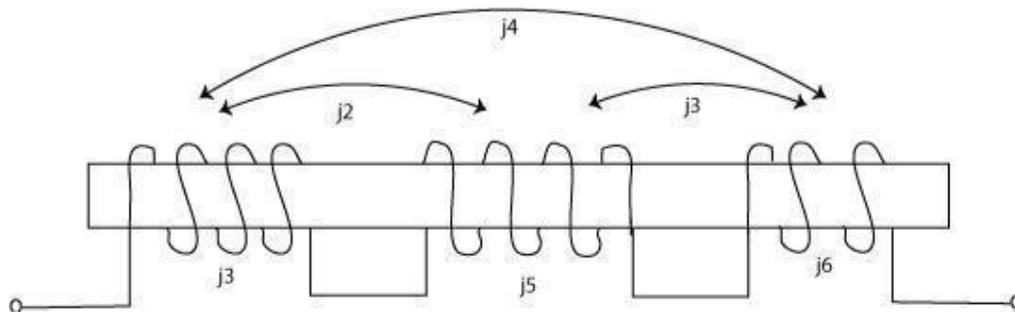
Solution: The dotted equivalent circuit is
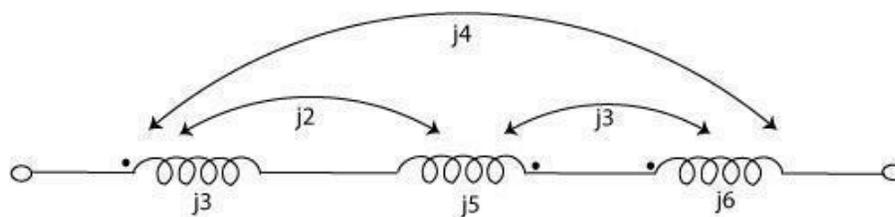
The equivalent inductive reactance is

j Xeq = j3 + j5 + j6 - 2 x j2 -2x j3+ 2x j4) = j14 - j2 = j12

Problem: Sketch the dotted equivale1lt circuit for the coupled coils shown in figure and find the equivalent inductance at the terminals AB. All coupling coefficie1lt are 0.5.
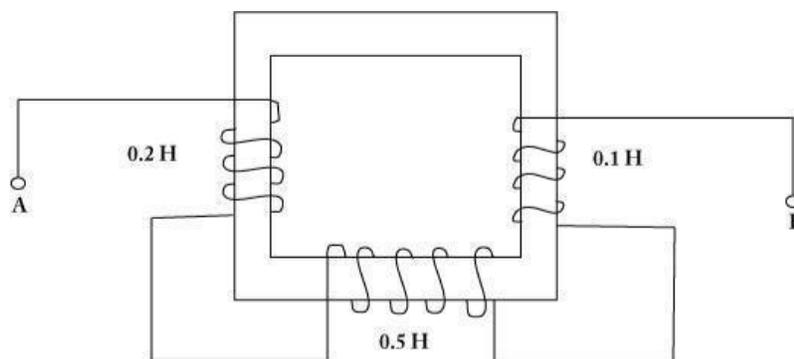
Solution: The dotted equivalent circuit is
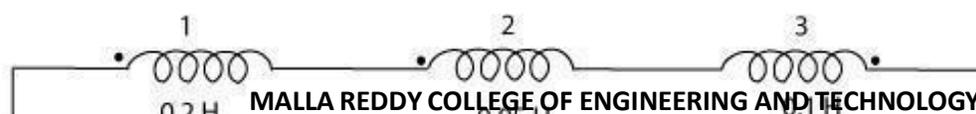
$M_{12} = 0.5\sqrt{0.2 \times 0.05} = 0.05$

$$= \sqrt{\quad \times \quad} =$$

$M_{23} \quad 0.5\sqrt{0.05 \ 0.1} \quad 0.0035$

$$= \sqrt{\quad \times \quad} =$$

$M_{31} \quad 0.5\sqrt{0.2 \ 0.1} \quad 0.0707$

Equivalent inductance between terminals A and B L_{AB} = 0.2 + 0.05+ 0.1 + 2(0.05 - 0.035 - 0.0707)

=0.2386H

Problem: For the given circuit as shown in figure

   a) Find the equivalent reactance
   b) Draw the transformer coupling circuit
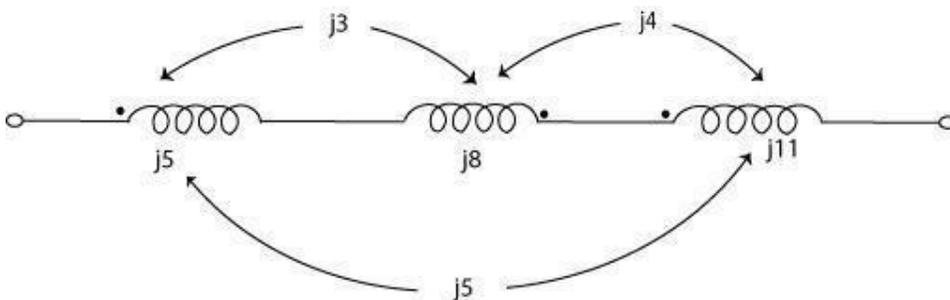
Solution

i)   The equivalent reactance is

j X_{eq} = j5 + j8 + j11 + 2(-j3 - j4 +j5) = j24 - j4 = j20

ii)   The T/F coupling circuit is
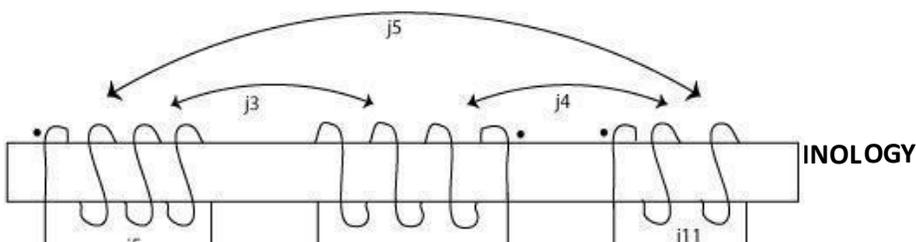
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Problem: Sketch the dotted equivalent circuit for the coupled coils shown in figure and find the current I?
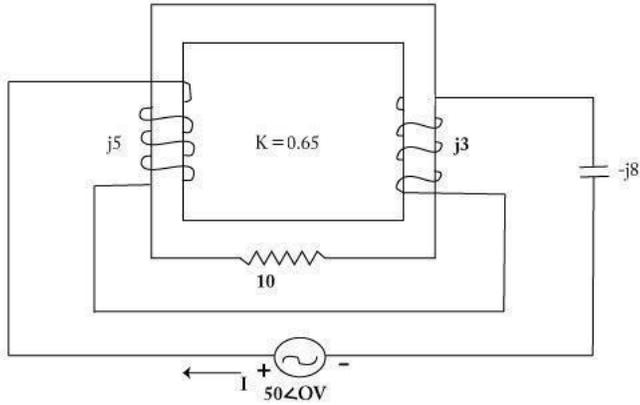
Solution: The dotted equivalent circuit is
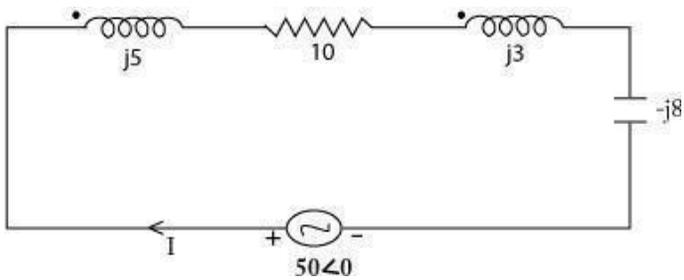
$$jX_M = jK \sqrt{X_{L1} \; X_{L2}} = j0.65\sqrt{5 \; 3} = j2.5$$

By applying KVL to the circuit

10I- j8I + (j5 + j3 + 2 x j2.5)I =50

I(10- j8 + j3) =50

I = 4.47 at an angle of 26.56 A